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Tracking Control of an Electro-Hydraulic Actuator System using LQR-ZPETC Controller

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Abstract: Electro-Hydraulic actuator (EHA) is a one type of application used in industry and building high performance of motion control process. Apparently, dealing with EHA behaviour is quite difficult and makes the controlling process complicated. Designing Linear Quadratic Regulator (LQR) controller as a feedback controller requires in selecting the weighting parameter Q and R. The result shows that the higher value of Q offers fast response and high stability by referring to the placement of closed-loop poles. However, the higher value of Q gives a higher error that can make the position performance of hydraulic actuator become worst. To overcome this problem, the feedforward controller is developed by implementing the zero-phase error tracking control (ZPETC). Simulation results showed that the LQR-ZPETC method was capable to produce satisfactory tracking performance.

Keywords: electro-hydraulic actuator, tracking performance, linear quadratic regulator, zero phase error tracking control

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1. INTRODUCTION

Electro Hydraulic Actuator (EHA) is widely implemented in industrial processes. This is because of its fast response, linear movement and accurate positioning of heavy load, thus making EHA one of the important tools for the industrial process [1]. These characteristics are the reasons why the hydraulic actuator is commonly used in industries and researches such as material handling [2], flight simulation [3], robotics [4], railway vehicle [5], valve for aircraft braking systems [6], active suspension system [7] because all of these applications rely on its control of the speed and position of the loads. Hydraulic offers many advantages such as good capability in positioning, fast and smooth response characteristic and also high-power density. Due to its capability in positioning, it has given a significant impact on modern equipment for position control application [8].

In terms of speed and direction, the original power source for the hydraulic actuator system is coming from the electric motor or the engine. These substances can be run at adjustable both in the constant speed which can drive the pump that can be driven at speeds varying by the large amounts of load. In addition, the hydraulic actuator also can be reversed instantly while the full motion without any damages [9]. The natural nonlinear properties of hydraulic cylinders give some obstacles, especially in identifying the suitable controller for position, motion and also the tracking control.

Nevertheless, dealing with EHA system can be the controlling process become difficult because of behaviours of EHA itself. The behaviour such as highly nonlinearities, uncertainties and time varying characteristics make the researchers concern about the controlling process [10]. To design the controller, a model of EHA behaviour needs to obtain first [11]. The EHA model for this paper was well-prepared by the previous researcher by using system identification method. This method was selected because it is less complicated and easier to apply to compare using the Physical Law method [12]. Before that, implementation of LQR required the changing of transfer function into the state space form. It is requiring A, B, C and D matrices as a system equation. ‘A’ matrix is the system or coupling matrix, ‘B’ as the input matrix, ‘C’ as the output matrix and lastly ‘D’ as a direct transition or feedforward matrix [13].

This paper discussing on implementation of a linear quadratic regulator (LQR) for non-minimum phase EHA system. LQR is one of optimal control which commonly used in academic study especially in feedback controllers for position tracking control. The LQR is designed as a feedback controller while implementation of feedforward is proposed in order to laminate the phase error that emerges during the tracking control process.

The rest of this paper is organized as follows. The plant background are presented in section 2. Section 3 presents the methodology. Section 4 presents the simulation results and discussion. Finally, conclusion is drawn in section 5.

2. PLANT BACKGROUND

Electro Hydraulic Actuator (EHA) becomes popular among academia especially in research field. It is because hydraulic actuators are rugged and suitable work for high-force applications. Furthermore, EHA can produce forces 25 times greater than a pneumatic cylinder of equal size. However, nonlinearities uncertainties and time varying characteristic of EHA behaviour will give a problem in controlling process. One of EHA plant that be used by previous research is bidirectional cylinder of EHA with 150mm strok length; 40mm bore size and 25mm rode size. EHA plant is injected by using a current range between 4-20mA and the input voltage applied was ±10V dc [14]. In this paper, the controlling scheme used was a pole placement method. This type of conventional control system applied and proved that the controller improves the reliability of the model based on tracking performance to the variation of reference input signal [1].

Moreover, EHA is also applied by discrete Proportional Integral Derivative (PID) controller in order to monitor the performance of the system. However, the system modelling has to obtain first by using the system identification method. The best correlation analysis of residual and best fit criterion is analysed to identify the adequate model to represent the EHA system [8]. The tracking in a real time system was used to verify these controllers. In order to improve the performance and position tracking, the system presents LQR as a feedforward controller. So, the implementation of both feedforward and feedback control scheme gives a better output performance more than classical PID controller.

The other controller was implemented into the EHA system was Fuzzy PID controller [15]. The controller develops in controlling the position variation of the EHA system. Behaviour of EHA system represent in the mathematical model by using the same method that is the system identification technique. Nevertheless, the output performance measured by comparing the controller design with the pole placement controller. The discussion stated that self-tuning Fuzzy PID produce better result compared pole-placement controller in term of Root Mean Squared Error (RMSE). Designing Linear Quadratic regulator required in identifying the value of parameter of weighting matrices Q and R. If the selecting of Q and R are not proper, then the positive definite solution will not exist and the controller designed not meet specified performance. The research paper said that by increasing the value of Q, the stability margin and bandwidth increase simultaneously. Furthermore, the increase of Q also gives a system more speedy and stable. The other parameter that affect the performance of the controller is the value of R. The decreasing value of R can give an effects on control input. It will decrease the required controlling input and causes an increase in the maximum feedback gain [16]. So, the correlation between the value of Q and R, are important in order to design LQR.

The plant used for this research is shown in Figure 1. It is the horizontal electro hydraulic actuator that consists of single-rod and double acting hydraulic cylinder. It driven by a direct servo valve Bosch Rexroth 4WREE6, 40lpm flow rate at 80 bar. The hydraulic cylinder has 63/30/300 (mm) in dimension. The position of the piston is measured by using 300mm draw wire sensor. The control and associated data acquisition is realized using National Instruments Data Acquisition Card (DAQ) [17].

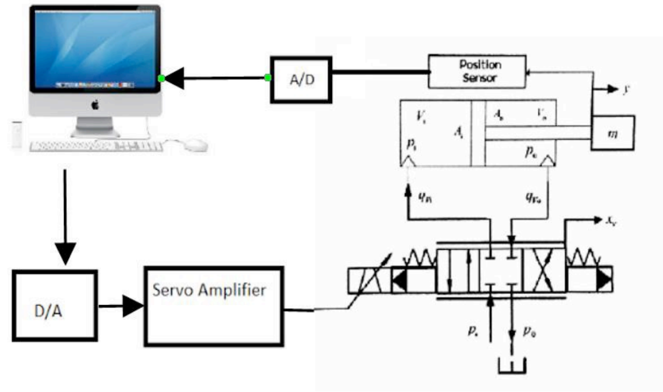

Figure 1. Electro-Hydraulic System

In this research, the non-minimum phase model is used in designing a perfect controller. The model was selected from the previous researcher. The non-minimum phase model represented in the discrete time transfer function is as follows;

$$G(z) = \frac{B_o(z^{-1})}{A_o(z^{-1})} = \frac{-0.3647z^2 + 0.4595z - 0.2004}{z^3 - 2.0411z^2 + 1.3932z - 0.3531} \tag{1}$$

The non-minimum phase model is known as a model that has one zero outside the unity circle. The location of poles and zeros of discrete transfer function model given in equation 1, can be represented by using pole-zero plot in Figure 2.

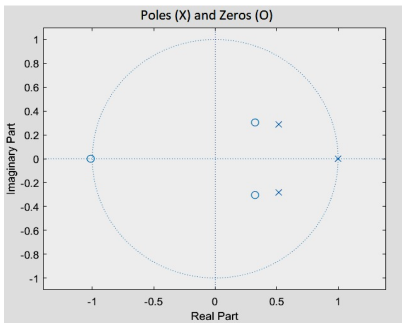

Figure 2. Pole-Zero plot

3. CONTROLLER

Figure 3 shows the implementation of the controller into EHA system. The details for each controller are stated in the following sections. Section 3.1 will explain about the implementation of LQR as a feedback controller while section 3.2 will explain about the ZPECT as a feedforward controller.

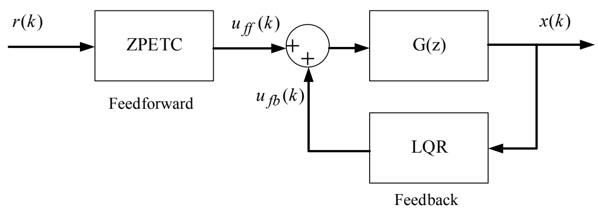

Figure 3. Proposed LQR-ZPETC controller

3.1 Linear Quadratic Regulator

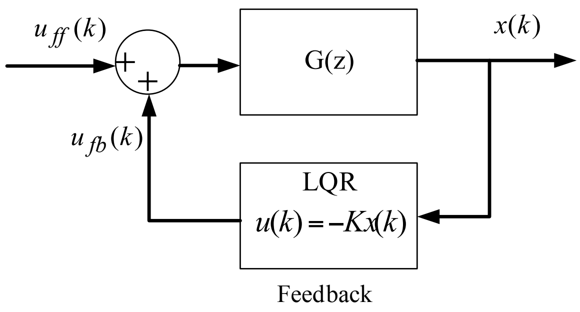

Figure 4. LQR feedback controller

Linear Quadratic Regulator is one of optimal control which has been used in many applications and widely developed by the academia and previous researchers. The function of the optimal control is a set of differential equations that describing the part of the control variables that minimize the cost function [18]. Besides that, optimal control also is a standard method for solving dynamic optimization problems and those problems are commonly expressed in continuous time. One of the advantages of using this control scheme is this method is well-known for its robustness towards uncertainties and disturbance. In order to reach perfect tracking control, the LQR feedforward controller switches to be the inverse of the closed-loop feedback system. From the model selected for this research, the LQR gain for the discrete-time model in equation (1) can be determined by minimizing the cost function as follows:

$$J = \sum x'Qx + u'Ru + 2x'Nu \tag{2}$$

The closed loop transfer function combines the Linear Quadratic Regulator can derive as follows:

$$u(k) = -Kx(k) \tag{3}$$
$$x(k+1) = (A - BK)x(k) \tag{4}$$
$$A_{cl} = A - BK \tag{5}$$
$$x(k+1) = A_{cl}x(k) + Br(k) \tag{6}$$
$$\frac{y(k)}{r(k)} = C(zI - A_{cl})^{-1}B = G_{cl}(z) \tag{7}$$

In designing the LQR controller, the discrete-time transfer function in (1) should be in the system matrix. The value of system matrix A and input matrix B is used to determine the value of LQR parameters like Q and R. Generally, the value of Q and R can be chosen as diagonal matrices or known as identify matrices. Based on the model of EHA system in equation (1), the system matrix A and input matrix B is used to design the LQR. By letting,

$$Q = C^T * C \quad \text{and} \quad R = 1$$

And minimizing the cost function, the LQR gain is:

$$K = [0.2591 \quad -0.2659 \quad 0.0976] \tag{8}$$

Then, the closed-loop system matrix can be shown as below:

$$A_{cl} = \begin{bmatrix} 1.7820 & -1.1273 & 0.2555 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{9}$$

Therefore, the closed-loop of transfer function for (1) can be represented as:

$$G_{cl}(z) = \frac{y(k)}{r(k)} = C(zI - A_{cl})^{-1}B$$
$$G_{cl} = \frac{0.3647z^2 - 0.4595z + 0.2004}{z^3 - 1.7820z^2 - 1.1273z + 0.2555} \tag{10}$$

3.2 Zero Phase Error Tracking Control

The tracking control system with two-degrees-of-freedom that consisting of feedback and feedforward controllers is given in Figure 3 as reported by Tomizuka [19] and R. Ghazali et.al [20]. In a tracking control system without the feedforward controller, the reference signal continuously varying and mixed with the closed-loop system dynamics, which made the function of feedback controller to be regulation against disturbance inputs. The feedforward controller is required such that the reference signal can be pre-shaped by the feedforward controller so that more emphasis on the frequency components that were not sufficiently handled by the feedback system can be provided [20]. Based on Figure 3 and equation (10), the function B_c (z^{-1}) can be factorized into minimum phase and non-minimum phase factors as given in equation (11). $B_C^+(z^{-1})$ denotes the minimum phase factor and $B_C^-(z^{-1})$ denotes the non-minimum phase factor.

$$B_c(z^{-1}) = B_c^+(z^{-1})B_c^-(z^{-1}) \tag{11}$$

Thus, from equation (10) the closed-loop transfer function can be represented by:

$$G_{cl}(z^{-1}) = \frac{z^{-d}B_C^+(z^{-1})B_C^-(z^{-1})}{A_C(z^{-1})} \tag{12}$$

Taking the inverse of closed-loop equation in equation (12), the ZPETC controller can be expressed by equation (13):

$$C(z^{-1}) = \frac{z^d A_c(z^{-1})B_c^-(z)}{B_+^c(z^{-1})[B_c^{-1}(1)]^2} \tag{13}$$

From equation (13), the ZPETC as reported in the literature [19 & 21] can be divided into three blocks as shown in Figure 5. The block diagram of feedforward ZPETC consists of the gain compensation filter, phase compensation filter and stable inverse.

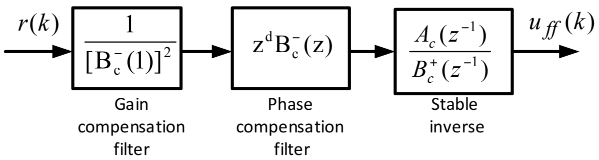

Figure 5. ZPETC controller structure

The feed-forward control can be design by letting the inverse of closed loop transfer function with LQR controller.

Based on equation (12), the direct inversion model represented by equation (14):

$$G_{ff}(z) = [G_{cl}(z)]^{-1} = \frac{A_c(z)}{B_c(z)} \quad (14)$$

$$G_{cl}(z^{-1}) = \frac{z^3 - 1.7820z^2 - 1.1273z + 0.2555}{0.3647z^2 - 0.4595z + 0.2004}$$

From the equation in Figure 5, one of the poles in the feedforward is outside unity circle, therefore equation (14) can represented in delay form as:

$$G_{ff}(z^{-1}) = \frac{A_c(z^{-1})}{B_c(z^{-1})} \quad (15)$$

$$= \frac{1 - 1.7820z^{-1} - 1.1273z^{-2} + 0.2555z^{-3}}{0.3647z^{-1} - 0.4595z^{-2} + 0.2004z^{-3}}$$

Based on equation (11), the numerator can be factorized, as presented in form as:

$$G_{ff}(z^{-1}) = \frac{A_c(z^{-1})}{B_c^+(z^{-1})B_c^-(z^{-1})} \quad (16)$$

$$G_{ff}(z^{-1}) = \frac{1 - 1.7820z^{-1} - 1.1273z^{-2} + 0.2555z^{-3}}{0.3647z^{-1}(1 - 1.6027z^{-1})(1 + 0.3429z^{-1})} \quad (17)$$

Based on equation (11),
$B_c^+(z^{-1})$ denotes the minimum phase factor and $B_c^-(z^{-1})$ denotes the non-minimum phase factor.

$$B_c^+(z^{-1}) = 0.3647z^{-1}(1 - 1.6027z^{-1}) \quad (18)$$

$$B_c^-(z^{-1}) = 1 + 0.3429z^{-1} \quad (19)$$

Based on Figure 5, the stable inverse can be represented in form as:

$$\frac{A_c(z^{-1})}{B_c^+(z^{-1})} \quad (20)$$

$$= \frac{1 - 1.7820z^{-1} - 1.1273z^{-2} + 0.2555z^{-3}}{0.3647z^{-1}(1 - 1.6027z^{-1})}$$

And based on the proposed controller, the phase compensation filter can be represented as following equation (21) where d=1

$$z^d B_c^-(z) = z(1 + 0.3429z) \quad (21)$$

And the gain compensation filter can be described as:

$$\frac{1}{[B_c^-(1)]^2} = \frac{1}{[1 + 0.3429(1)]^2} = 0.55451 \quad (22)$$

4. RESULT AND DISCUSSION

A. LQR Implementation

The initial requirement for designing Linear Quadratic Regulator is dependent on choices of weighting matrices Q and R. If Q and R are not properly selected then the designed controller for both feedback and feedforward control will not meet the exact performance of the EHA system. In general, the value can be chosen as identity matrices. The other choice is to consider Q as $C^T C$. Design LQR controller for EHA system describe by (1) with a range of weighting matrices as follows;

$$Q = 0.1(C^T * C) \quad \text{to} \quad 100(C^T * C) \quad (21)$$

$$R = 0.1 \quad (22)$$

There are specific reasons why the value of R selected is small. This is because the large value of R will give a smaller the value of input vector but it requires a large value of the corresponding column of Q. The increasing of R also will penalize more on the control input. This section will explain an effect for different values of Q while a fixed value of R. The parameters of feedback gain K, eigenvalues and Root Mean Squared Error (RMSE) of closed-loop system are tabulated in Table 1.

Table 1 shows that the feedback gain is directly proportional to the value of weighting matrices Q. If the value of Q is high, the feedback gain K also simultaneously increases. Moreover, it can be seen that the closed-loop poles are placed more deeply into the left half-plane which makes the EHA system more stable. In terms of error, the large value of Q gives a large error. This error is the difference between the output of the feedback controller and the input trajectory applied to the system.

Table 1. Q and RMSE performance

Weighting Matrices	K	Eigenvalues	RMSE
$Q = 0.1(C^T * C)$	[0.2591 -0.2659 0.0976]	0.6627 + 0.0000i 0.5597 + 0.2691i 0.5597 - 0.2691i	6.01499
$Q = 1(C^T * C)$	[0.5415 -0.5686 0.2273]	0.2415 + 0.0000i 0.6291 + 0.3538i 0.6291 - 0.3538i	14.0059
$Q = 10(C^T * C)$	[0.7374 -0.7924 0.3294]	0.0433 + 0.0000i 0.6302 + 0.3861i 0.6302 - 0.3861i	18.0766
$Q = 100(C^T * C)$	[0.7763 -0.8380 0.3505]	0.0048 + 0.0000i 0.6300 + 0.3902i 0.6300 - 0.3902i	18.6278

Figure 6 shown the output response of LQR controller. After selected the values of weighting matrices Q for LQR, the system was tested by using unit step input to observe the output of the tracking control performance. It can be seen that the large values of Q will improve the transient performance by giving small values of settling time, rise time and steady-state error. It can be seen that the system showed a faster response and good stability. From the Figure 6, the tracking output was poor if using small values of weighting matrix Q. Poor tracking performance is observed and makes the system difficult to control. Also from the result, the steady state error was poor if increase the values of Q. It shows that the phase lag problem occurs during the tracking control. This error shows significant different between the desired trajectory and the actual output.

B. LQR-ZPETC Tracking Performance

By introducing the ZPETC controller, the phase lag can be reduced in the tracking control. The tracking performance using LQR and ZPETC is shown in Figure 7. The results obtained with the proposed LQR-ZPETC controller are ~~is~~ better as compared to LQR controller where the proposed controller produced more accurate results to reach the desired trajectory. And the performance criteria for the tracking process are tabulated in Table 2.

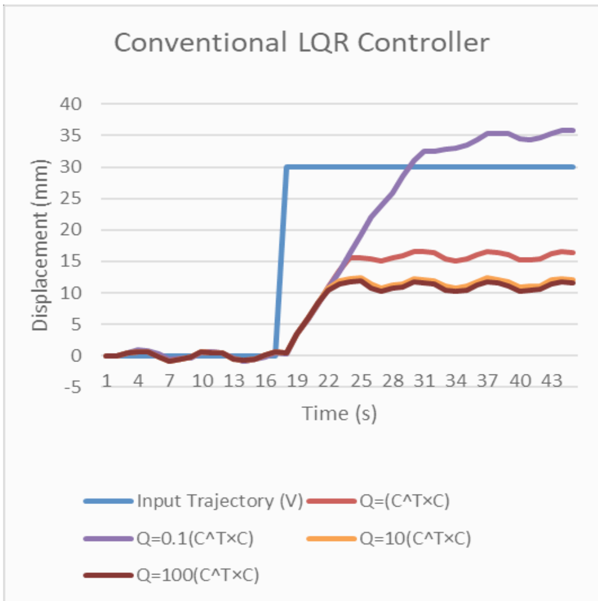

Figure 6. Conventional LQR controller

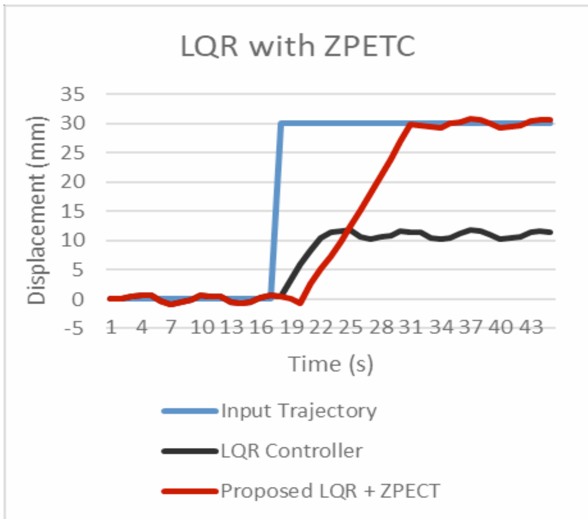

Figure 7. Tracking performance LQR-ZPETC Controller

Table 2. RMSE performance criteria

Weighted Matrix (Q)	LQR	LQR-ZPETC
$Q = 0.1(C^T \times C)$	6.0149	4.2515
$Q = (C^T \times C)$	14.0059	4.1875
$Q = 10(C^T \times C)$	18.0766	4.1961
$Q = 100(C^T \times C)$	18.6278	4.1974

Based on Root Mean Square Error (RMSE) tabulated in Table 2, the error analyses, control efforts and observation on the tracking performance, the LQR-ZPETC provides more convenient and better performance in trajectory tracking control and ensured that the control system is under a stable condition.

5. CONCLUSION

In this study, the finding of LQR weighting matrices Q and R in the first step of a truncated iteration is used for computing the exact gain matrix for output feedback.

Simulation results showed that the LQR-ZPETC method was capable to produce satisfactory tracking performance.

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