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A Quasi-moment-method Modelling of Energy Demand Forecasting

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Abstract: This paper presents a novel approach to the modelling of electrical energy demand forecasting, based on the Quasi-Moment-Method (QMM). The technique, using historical energy consumption/demand data, essentially calibrates nominated 'base' models (in this case, nominal Harvey and Autoregressive models) to provide significantly better performing models. In addition to the novelty of the use of QMM, the paper identifies hitherto unreported singularities of the generic Harvey / logistic model, through which a simple, but remarkably pivotal modification is proposed, prior to the model's use as base model in QMM calibration schemes. The treatment of the 'Harvey singularities' informed a similar and equally significant modification of the Autoregressive model utilized in the paper. For the purposes of validation and performance evaluation, computational results due to the QMM models are compared with corresponding results reported in three different journal publications, which utilized the Harvey and Autoregressive models in conventional regression schemes. And in terms of the usual model performance metrics (including Mean Absolute Percentage Error (MAPE) and Root Mean Square Percentage Error (RMSPE)), the results very clearly demonstrate the superiority of the QMM models for both energy demand prediction and forecasting. As representative examples, a QMM-calibrated Harvey model recorded an RMSE value of 495.45dB for total energy consumption prediction, as against 618.60dB obtained for the corresponding nominal Harvey model: and for the Autoregressive case, RMSE was obtained as 131.35dB for QMM model's prediction of peak load demand, compared with the 173.40dB due to the nominal model.

Keywords: Autoregressive Model, Energy Demand Forecasting, Harvey Singularity, Quasi-Moment-Method, Time Series.

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1. INTRODUCTION

In a recent, comprehensive, and arguably exhaustive review of the literature on energy demand forecasting, Verwiebe et al [1], observed that it is a generally held position that energy policies are increasingly being influenced by a requirement for simultaneously reliable, climate-change-aware, and cost-effective energy systems: with the consequence that planning processes routinely utilize models, for which prediction of future demand is a critical input. It is not surprising therefore to find that research into energy demand modelling is continuously being motivated by the need identified in the foregoing, and has produced quite a few approaches; many of which are specific to global demand forecasting, [1], some others, to demand prediction in regional and national settings, [1], [2], and a few others, solely to energy planning needs, [3]. These modelling techniques, which vary markedly in analytical complexity and forecasting accuracy, fall into the five broad categories referred to [1], as statistical methods (including regression and time series analysis), machine language approaches (typified by analysis based on Artificial Neural Network (ANN), K-Nearest Neighbour (KNN) and Support Vector Machine (SVM) algorithms); metaheuristic methods (involving evolutionary algorithms such as Particle Swarm Optimization (PSO), Genetic Algorithm (GA), and firefly algorithms; stochastic/ fuzzy/ Grey systems (with the fuzzy time series, Adaptive Neuro-Fuzzy Inference Systems (ANFIS), and Hidden Markov Model as common examples): and the engineering based techniques, which utilize the laws of physics.

Typical implementations of these techniques, in the case of the machine learning approaches, are exemplified by the contributions of [4], in which the use of random forest algorithms to facilitate efficient processing of large volumes of data provided a model for the prediction of household energy consumption. Another example is offered by the two-stage scheme utilized in combination with the Long Short-Term Method (LSTM) and a variation of the moving average method, for the purposes of predicting potential highest demand peaks, Jiménez et al.[5], Chou and Truong [6], in a metaheuristic approach, optimized the performance of a time series model through the use of non-linear machine learning models involving least squares support vector regression; whilst the metaheuristic framework developed by Azadeh and Faiz [7], for the prediction of household energy consumption, combined multi-layer perception, conventional regression, and the 'Design of Experiments (DoE)' algorithms. For the class of stochastic modelling techniques, a typical implementation example is provided by Patidar et al [8], who investigated the effectiveness of two stochastic modelling schemes (one, a hybrid of time series deseasonalization and a single hidden Markov model; and the other, a time series deseasonalized Autoregressive Integrated Moving Average (ARIMA) model) in the generation of synthetic energy demand characteristics. And another is the contribution by Ahmadi et al [9], whose stochastic model, utilized for long term energy demand forecasting, combined scenario analysis with a Bayesian approach. The model proposed by Rehfeldt et al [10], which is representative of the engineering-based techniques, describes the 'bottom-up' characteristics of this class, as applied to the solution of the problem of disaggregating energy end-use balance of interest to industrial heating and cooling processes.

In the case of the category of statistical techniques, the schemes presented by Li [11], for the investigation of the relative merits and demerits of conventional regression and time series analysis as models for forecasting energy consumption, represents a good example. Another is the contribution by Ismail et al [12], in which the impact of holidays, weather conditions, and monthly seasonality on daily and monthly energy demand was evaluated with the use of a multiple regression model; and which also utilized a time series prediction model, for the purposes of model performance comparison.

An excellent summary of the strengths and weaknesses of each of the categories of energy demand modelling discussed in the foregoing, as widely held in the literature, is provided in a tabular format, in [1]. And according to this account, a common advantage shared by the machine learning, metaheuristics, and statistical categories is that of ease in implementation; with the machine learning techniques being assigned the additional merits of high prediction accuracy and the ability to effectively handle non-linear relationships. Also, both the statistical and metaheuristic techniques share the advantage of having the 'white-box' property, in addition to which statistical techniques are credited with the merits of low data input requirements and ability to efficiently account for uncertainties in input data. Other advantages attributed to the metaheuristic methods include easy hybridization with other models and ability to handle different types of demand forecasting problems. The main advantage credited to the engineering based techniques is that they are particularly suitable for the simulation of energy demand profiles of interest to scenarios associated with disruptions for which historical records are unavailable.

On the flip side, machine learning techniques have the disadvantage of a 'black-box' property, as well as the possibilities (which they share with statistical methods) of overfitting and getting stuck in local minima. The statistical methods are assigned the additional demerits of the limitation imposed by correlated independent variables and inability to efficiently handle extreme events and outliers in historical data. Metaheuristic techniques require additional knowledge for implementation, and are often plagued with low convergence rates. One notable disadvantage of the stochastic modelling methods is their outputs are typically available as probabilistic or fuzzy expressions, and that of the engineering based modelling methods is that they are necessarily knowledge- and data-intensive.

Of these modelling techniques, the statistical techniques are widely regarded as the simplest and next only to the computational intelligence methods in prediction accuracy, [1], [12]. Furthermore, in the class of statistical techniques, the time series analysis has been demonstrated in several comparative investigations, [11], [13], to be better performing than the regression approach. Although these investigations also reported results that indicated that the ARIMA model is the best of the Time Series Analysis (TSA) models, they pointed out that the method is analytically relatively demanding and is not applicable in all cases, because the model's validity requires several decades of historical data concerning some explanatory variables, Mohammed and Mohamed [14]. A few other comparative studies [15], [16], concluded that in the class of time series analysis techniques, the Harvey models perform better than the others; and in particular, the results published by Mohamed and Bodger [17], [18], very clearly suggest that the Harvey model has better performance metrics than the ARIMA model.

It is the main objective of this paper, to introduce the Quasi-Moment-Method (QMM) [19] as a simple, very easy to implement, and remarkably efficient tool for energy demand forecasting modelling. To this end, and for reasons described in the foregoing review of the literature, the generic Harvey model is selected as main candidate for investigations. Analysis presented in the paper reveals, for the first time to the best of our knowledge, that the Harvey model is characterized by an inherent singularity (here referred to as the 'Harvey singularity'), which when accounted for by the QMM modelling process, leads to a QMM model, whose performance metrics are much better than those of the base models from which it derives. The validity of the QMM energy demand forecasting modelling process is established through a performance comparison of its metrics (including Mean Absolute Percentage Error (MAPE), Mean Absolute Error (MAE), and Root Mean Square Percentage Error (RMSPE)) with corresponding metrics due to three other time series modelling processes (utilizing least squares regression) reported in the literature, [15], [16], [18]. Another novelty introduced in the paper is that two alternatives for specifying time (denoted by 't') were considered in the paper. One alternative utilized $t = 1, 2, \ldots, T$, as in [14] and [15], and the other, $t = 1943, 1944, \ldots, 1999$, as utilized in Mohamed and Bodger [18]. Computational results suggest that either alternative gives about the same outcome, and in each case, the QMM model consistently outperformed the other models.

Details of the theoretical basis for the QMM modelling are presented section 2 of the paper, and the computational results due to the QMM models are discussed in section 3, which also discusses the outcomes of the comparisons of the performance metrics. The main conclusions due to findings in the paper are highlighted in section 4, the paper's concluding section.

2. ANALYSIS

The Quasi-Moment-Method is an empirical modelling technique originally introduced by Adekola et al [19] as a tool for the modelling of radiowave propagation path loss in wireless communications, whose basic properties and ubiquity in that connection have since been firmly established through a number of publications, including [20], [21], and [22].

For the description of QMM modelling of energy demand forecasting of interest here, a good starting point is the generic Harvey model given (Mohamed and Bodger [17], [18]), as

$$\log_e(y_t) = \rho \log_e(Y_{t-1}) + \delta + \gamma t, \tag{1}$$

in which the parameter denoted by 'ρ' (and here referred to as the 'Harvey model exponent) defines different forms of the Harvey model according to

$$\rho = \begin{cases} -1, & \text{Harvey Logistic model,} \\ 0, & \text{simple exponential model;} \end{cases} \tag{1a}$$

and for all other values of ρ, the Harvey model. The variable Y_{t-1} appearing in Equation (1) represents the actual consumption (demand) data recorded for the year immediately preceding the year represented by 't', and $y_t = Y_t - Y_{t-1}$ is the difference between the two, δ and γ are constants conventionally determined through the regression of $\log_e\left(\frac{y_t}{Y_{t-1}^{\rho}}\right)$ on t, when Equation (1) is recast in the form given [17], as

$$\log_e\left(\frac{y_t}{Y_{t-1}^{\rho}}\right) = \delta + \gamma t. \tag{2}$$

Because the natural logarithmic function is undefined for arguments less than or equal to zero, and since cases of $Y_t < Y_{t-1}$ (for some values of 't') are common with historical electrical energy consumption records, it follows that the generic Harvey model, as defined by Equations (1) and (2), is characterized by a singularity (the 'Harvey singularity'), which is here being explicitly identified for the first time, to the best of our knowledge. The other singularity due to $y_t = 0$ is treated by perturbing the affected data to make y_t very slightly different from zero.

Towards a QMM solution and in order to account for the Harvey singularity, Equation (2) is modified to read

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{\rho}}\right) = \delta + \gamma t, \tag{3}$$

from which it follows that

$$Y_t = \begin{cases} Y_{t-1} - Y_{t-1}^{\rho}\left(e^{(\delta+\gamma t)}\right), & y_t < 0 \\ \text{or} & t = 2,...,T \\ Y_{t-1} + Y_{t-1}^{\rho}\left(e^{(\delta+\gamma t)}\right), & y_t > 0 \end{cases} \tag{4}$$

An interesting possibility is offered by Equation (4) when δ and γ are so specified that

$$\delta + \gamma T << 1; \tag{5}$$

for then, Equation (4) assumes an approximate form according to

$$Y_t \simeq \begin{cases} Y_{t-1} - Y_{t-1}^{\rho}(1 + \delta + \gamma t), & y_t < 0 \\ \text{or} & t = 2,...,T, \\ Y_{t-1} + Y_{t-1}^{\rho}(1 + \delta + \gamma t), & y_t > 0 \end{cases} \tag{6}$$

a series approximation particularly suitable for QMM modelling, and which derives from Equation (4) through the use of the well-known fact that when $x << 1,\ e^x \simeq 1 + x.$

It is also a matter of interest to remark that the treatment of the Harvey singularity as described by Equation (6) informed a modification of the classical autoregressive (AR) time series model, prior to QMM modelling. In this case, it is first noted that the AR model is conventionally specified [15], [16], as

$$Y_t = \beta_0 + \beta_1 Y_{t-1}, \tag{7}$$

which inherently assumes that $y_t > 0, \forall t$. The associated remedy proposed in this paper is to first define $\beta_1 = 1 + \sigma$, such that Equation (7) modifies to

$$|y_t| = \beta_0 + \sigma Y_{t-1}, \tag{8}$$

or, equivalently,

$$Y_t = \begin{cases} Y_{t-1}(1-\sigma) - \beta_0, & Y_t < Y_{t-1} \\ Y_{t-1}(1+\sigma) + \beta_0, & Y_t > Y_{t-1} \end{cases}, t = 2, \ldots, \text{T} \tag{8a}$$

The QMM solution described in the section 2.1 uses historical energy demand data to calibrate models defined by Equations (3) (and its series approximation) and (8), to develop demand forecast models according to correspondingly calibrated versions of Equations (4), (6), and (8).

2.1 Quasi-Moment-Method Models

The first step in the implementation of the QMM algorithm in the case of the generic Harvey model is, with reference to Equation (3), to make the following definitions:

$$X = \log_e\left(\frac{|y_t|}{Y_{t-1}^{\rho}}\right), \tag{9}$$

$$\varphi_1 = \delta, \tag{9a}$$

and

$$\varphi_2 = \gamma t; \tag{9b}$$

so that the desired QMM model emerges in this case, as

$$X = \alpha_1\varphi_1 + \alpha_2\varphi_2\ . \tag{10}$$

The unknown calibration coefficients are then obtained through the simple matrix processes of inversion and multiplication prescribed as

$$\begin{pmatrix}\alpha_1 \\ \alpha_2\end{pmatrix} = \begin{bmatrix}\langle\varphi_1,\varphi_1\rangle & \langle\varphi_1,\varphi_2\rangle \\ \langle\varphi_2,\varphi_1\rangle & \langle\varphi_2,\varphi_2\rangle\end{bmatrix}^{-1}\begin{pmatrix}\langle\varphi_1,X\rangle \\ \langle\varphi_2,X\rangle\end{pmatrix}. \tag{11}$$

The inner product quantity denoted by $\langle\bullet,\bullet\rangle$, is given Adekola et al (2021), for two functions φ_m and φ_n, by

$$\langle\varphi_m,\varphi_n\rangle = \sum_{t=2}^{T}\varphi_m(t)\varphi_n(t), \tag{12}$$

and in particular, with respect to Equation (11),

$$\langle \varphi_m, X \rangle = \sum_{t=2}^{T} \varphi_m(t) X \quad . \tag{13}$$

With the calibration coefficients available from Equation(11), the QMM model derived from the generic Harvey model gives the demand prediction for year 't' (denoted by $\tilde{Y}_t$) as

$$\tilde{Y}_t = \begin{cases} Y_{t-1} - Y_{t-1}^{\rho}\left(e^{(\alpha_1\delta + \alpha_2\gamma t)}\right), & y_t < 0 \\ \text{or} & t = 2, ..., T \\ Y_{t-1} + Y_{t-1}^{\rho}\left(e^{(\alpha_1\delta + \alpha_2\gamma t)}\right), & y_t > 0 \end{cases} \tag{14}$$

A similar procedure for the series approximation 'base' model defined by Equations (5) and (6) yields

$$\tilde{Y}_t \simeq \begin{cases} Y_{t-1} - Y_{t-1}^{\rho}(\alpha_1 + \alpha_2\delta + \alpha_3\gamma t), & y_t < 0 \\ \text{or} & t = 2, ..., T, \\ Y_{t-1} + Y_{t-1}^{\rho}(\alpha_1 + \alpha_2\delta + \alpha_3\gamma t), & y_t > 0 \end{cases} \tag{15}$$

provided, in this case, that

$$\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{pmatrix} = \begin{bmatrix} \langle\varphi_1,\varphi_1\rangle & \langle\varphi_1,\varphi_2\rangle & \langle\varphi_1,\varphi_3\rangle \\ \langle\varphi_2,\varphi_1\rangle & \langle\varphi_2,\varphi_2\rangle & \langle\varphi_2,\varphi_3\rangle \\ \langle\varphi_3,\varphi_1\rangle & \langle\varphi_3,\varphi_2\rangle & \langle\varphi_3,\varphi_3\rangle \end{bmatrix}^{-1} \begin{pmatrix} \langle\varphi_1, X\rangle \\ \langle\varphi_2, X\rangle \\ \langle\varphi_3, X\rangle \end{pmatrix} \tag{16}$$

with

$$\{\varphi_k\}_{k=1}^{3} = \{1, \delta, \gamma t\}, \tag{17}$$

and

$$X = \frac{|y_t|}{Y_{t-1}^{\rho}} \tag{18}$$

Finally, when the AR model prescribed by Equations (8) and (8a) is subjected to QMM calibration, the response is obtained as

$$\tilde{Y}_t = \begin{cases} Y_{t-1}(1 - \alpha_2\sigma) - \alpha_1\beta_0, & Y_t < Y_{t-1} \\ Y_{t-1}(1 + \alpha_2\sigma) + \alpha_1\beta_0, & Y_t > Y_{t-1} \end{cases}, t = 2, \ldots, \text{T} \tag{19}$$

The calibration coefficients in this case derive from expressions similar to those of Equations (11) and (12), with $\varphi_1 = \beta_o$. $\varphi_2 = \sigma$, and $\text{X}_t = |y_t|$.

3. RESULTS AND DISCUSSION

In order to establish the validity of the QMM models developed in the foregoing section, historical fit and forecasting results are compared with those due to the classical Harvey and autoregressive models as available from three different publications; one concerning energy consumption in New Zealand, [18], and the others, energy demand scenarios in Nigeria, (Okakwu et al [15]; and Ogungbemi et al [16].

3.1. QMM Performance Evaluation

Three different energy consumption scenarios, namely, 'domestic', 'non-domestic', and 'total', were considered in [18], for the purposes of evaluating the forecasting abilities of the Harvey models. Using data available from [18] for QMM modelling as described in section 2, corresponding historical fits and forecasting profiles were obtained, and their performance metrics compared with those reported in [18] , for the classical Harvey models. In each case, three variations of the generic Harvey model were considered as base models: first is the 'series approximation' model defined by

$$\frac{|y_t|}{Y_{t-1}^{\rho}} = 1 + 0.001 + 0.0001t, \quad t = 2, \ \ldots, \text{T}, \tag{20}$$

in which $\delta = 0.001$ and γ=0.0001 have been so selected as to ensure that the series approximation to the associated exponential function applies. Others are the Harvey logistic and Harvey models specified, in the case of domestic consumption, through Equations (11) and (14) of [18] as

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{2}}\right) = 150.86 - 0.083t, \tag{21}$$

and

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{0.60}}\right) = 35.44 - 0.018t, \tag{22}$$

respectively. Corresponding expressions for the non-domestic consumption case are

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{2}}\right) = 145.79 - 0.086t, \tag{23}$$

and

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{1.29}}\right) = 57.46 - 0.032t, \tag{24}$$

as prescribed for the Harvey logistic and Harvey models, respectively, by Equations (12) and (15) of Mohamed and Bodger [18]. The Harvey logistic and Harvey base models for total consumption are specified by Equations (13) and (16) of [18] as

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{2}}\right) = 145.60 - 0.081t, \tag{25}$$

and

$$\log_e\left(\frac{|y_t|}{Y_{t-1}^{1.08}}\right) = 50.27 - 0.028t, \tag{26}$$

respectively.

3.1.1. Calibration with Domestic Consumption Data from [18]

Outcomes of the calibration of the base models given by Equations (20), (21), and (22) with the domestic consumption data of Mohamed and Bodger [18] resulted in the QMM models defined by the calibration coefficients displayed in Table 1.

Examples of the QMM models defined by the parameters of Table 1 include

$$\tilde{Y}_t \simeq \begin{cases} Y_{t-1} - Y_{t-1}^{0.8}(0.0202 + 441.9118\delta - 74.6676\gamma t), & y_t < 0 \\ \text{or} & t = 2, ..., T, \\ Y_{t-1} + Y_{t-1}^{0.8}(0.0202 + 441.9118\delta - 74.6676\gamma t), & y_t > 0 \end{cases} \tag{27}$$

Table 1. Parameters of QMM Prediction Models due to calibration with Domestic Consumption Data [18]

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	0.75	0.0219	685.3531	-112.4310
	0.80	0.0202	441.9118	-74.6676
	0.85	0.0190	281.8347	-49.3052
	0.90	0.0175	177.7896	-32.3889
	1.02	0.0118	56.5990	-11.6046
Harvey Logistics	2.00	-0.0682	0.8763	N/A
Harvey	0.60	0.0069	0.7173	N/A

for the series approximation base model of Equation (6), when $\rho = 0.8$, δ and $\gamma = 0.001$ and 0.0001, respectively; with the calibration coefficients identified by the numerals in magenta colored fonts. Another is the QMM-Harvey logistic model, which according to the parameters of the third row of Table 1 and Equation (14), is given

$$\tilde{Y}_t = \begin{cases} Y_{t-1} - Y_{t-1}^2\left(e^{(-0.0682\delta + 0.8763\gamma t)}\right), & y_t < 0 \\ \text{or} & t = 2,\ldots,T \\ Y_{t-1} + Y_{t-1}^2\left(e^{(-0.0682\delta + 0.8763\gamma t)}\right), & y_t > 0 \end{cases} \tag{28}$$

For Equation (28) the values of δ and γ are 150.86 and -0.083, respectively.

Profiles of the domestic consumption predicted by the QMM models defined by the parameters of Table 1 are displayed in Fig. (1), from which, it is readily observed that all the QMM models have similar historical fit profiles.

The prediction (historical fit) performances of the QMM models are described by the metrics displayed in Table 2. And as can be seen from the profiles of Fig. (1) and the metrics of Table 2, all the QMM models have about the same prediction accuracy characteristics, though with the series-approximation-based models generally performing better than those that derived from the conventional forms of the Harvey models. The metrics also suggest that MPE, MAE, and RMSE increase slightly with increasing values of ρ; though both MAPE and RMSPE (widely regarded as prediction accuracy measures of preference, Verwiebe et al [1] improved with increasing ρ up to $\rho = 0.85$, after which their values increased slightly.

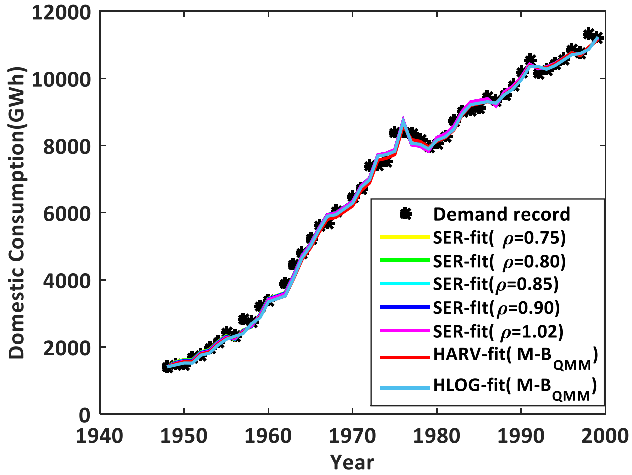

Figure 1. Profiles of historical domestic energy consumption and corresponding predictions due to the QMM models of Table 1.

Table 2. Prediction performance metrics for the QMM models defined by Table 1

METRIC/ MODEL	**MPE**	**MAE**	**MAPE**	**RMSE**	**RMSPE**
Series Approx. ρ = 0.75	36.53	150.12	2.93	186.67	3.92
Series Approx. ρ = 0.80	37.90	150.59	2.92	187.71	3.89
Series Approx. ρ = 0.85	39.56	151.05	2.91	188.85	3.88
Series Approx. ρ = 0.90	41.30	151.87	2.92	190.07	3.88
Series Approx. ρ = 1.02	46.28	153.72	2.94	193.27	3.93
Harvey	96.35	152.82	3.01	204.58	4.31
Harvey. Logistic	84.83	157.61	3.04	206.45	4.45

3.1.2. Calibration with Non Domestic Consumption Data from [18]

The calibration of the models defined by Equations (20), (21), and (22) with the non-domestic consumption data of Mohamed and Bodger [18] yielded the QMM models defined by the calibration coefficients of Table 3.

Table 3. Parameters of QMM Prediction Models due to calibration with Non-Domestic Consumption Data, [18]

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	0. 75	0.9482	71.6525	-124.7126
	0.80	0.3027	370.7160	-86.0235
	0.85	0.1625	278.7993	-58.3690
	0.90	-0.2026	489.8477	-39.0758
	1.02	0.0567	43.4935	-142.2956
Harvey Logistics	2.00	-0.0598	1.2223	N/A
Harvey	0.60	-0.0737	1.4491	N/A

These parameters define QMM energy consumption models in the same manner as described through the examples of Equation (28) and (29). Profiles of the non-domestic energy consumption predicted by the models are displayed in Fig. (2) and their associated prediction accuracy metrics are shown in Table 4.

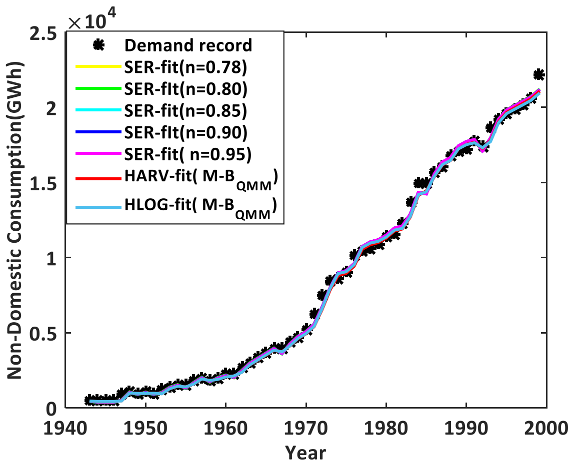

Figure 2. Comparison of energy consumption predicted by QMM models of Table 3 with actual consumption records [M B4]

Table 4. Prediction performance metrics for the QMM models defined by Table 3

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSP E
Series Approx. ρ = 0.75	83.56	242.58	5.00	339.27	8.57
Series Approx. ρ = 0.80	84.48	242.03	4.86	340.45	8.38
Series Approx. ρ = 0.85	85.82	242.46	4.82	341.87	8.25
Series Approx. ρ = 0.90	87.49	243.48	4.81	343.53	8.19
Series Approx. ρ = 1.02	92.09	247.91	4.91	348.36	8.57
Harvey	182.40	254.83	5.41	374.09	8.38
Harvey. Logistic	172.3	254.92	5.29	376.70	8.25

The model historical fit accuracy metrics of Table 4 clearly follow the same trends as described for the corresponding metrics of Table 2; though in this case all the metrics are significantly poorer than to those due to domestic consumption prediction. This is probably due to the relatively very low levels of non-domestic consumption in the World War II years between 1943 and 1945.

3.1.3.Calibration with Total Consumption Data from [18]

The model calibration coefficients obtained from the calibration of the base models of section 2 with the total energy consumption data available from Mohamed and Bodger [18], are displayed in Table 5.

Table 5. Parameters of QMM Prediction Models due to calibration with Total Consumption Data, [18]

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	0. 75	-0.0482	142.2470	-12.2470
	0.80	0.0107	81.5169	-11.5169
	0.85	0.1361-	-42.5034	-12.1528
	0.90	0.0204	74.1870	-12.6736
	1,02	0.0571	37.9303	-13.4946
Harvey Logistics	2.00	-0.0086	0.9124	N/A
Harvey	0.60	-0.0549	0.7488	N/A

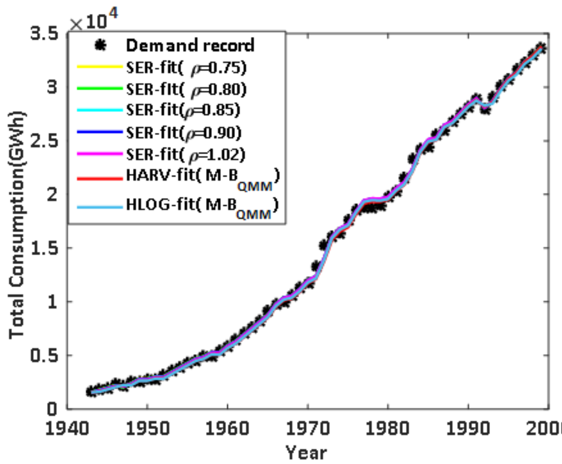

Figure 3. Energy consumption predicted by QMM models of Table 5 compared with actual total consumption records of [M B4]

And with the corresponding QMM models defined according to Equation (28) and (29) as earlier described in section 2, the associated historical fit curves are displayed in Fig. (3). It is apparent from the profiles of Fig. (3) that the QMM models developed with the use of total energy consumption in the calibration process have significantly better prediction accuracy metrics than those due to calibration with domestic and non-domestic energy consumption data.

Table 6. Prediction performance metrics for the QMM models defined by Table 5

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSPE
Series Approx. ρ = 0.75	20.09	245.70	2.42	362.56	3.62
Series Approx. ρ = 0.80	-0.04	246.04	2.38	360.04 8	3.52
Series Approx. ρ = 0.85	3.18	247.21	2.37	361.06	3.49
Series Approx. ρ = 0.90	6.16	249.18	2.40	362.30	3.48
Series Approx. ρ = 1.02	17.61	254.98	2.52	366.61	3.56
Harvey	173.39	302.86	3.28	412.20	4.40
Harvey. Logistic	160.28	309.93	3.36	410.67	4.52

Nonetheless, the variations with ρ, of all the metrics of Table 6 follow the trend described earlier for the domestic and non-domestic consumption cases, though there is the notable instance of MPE being close to zero for the series approximation model, when $\rho = 0.8$.

3.2. Forecasting Accuracy

In order to evaluate the forecasting accuracy of the QMM models, the base models of section 2 were first calibrated with total consumption data covering 1970 to 1990. The QMM models so obtained were then utilized for forecasting total energy consumption from 1991 to 1999, according to the schemes described as follows.

$$\hat{Y}_t = \hat{Y}_{t-1} + \hat{Y}_{t-1}^{\rho}\left(e^{(\alpha_1\delta+\alpha_2\gamma t)}\right), \quad t = 2,\ldots,T, \tag{29}$$

for the Harvey models, and

$$\hat{Y}_t = \hat{Y}_{t-1} + \hat{Y}_{t-1}^{\rho}\left(\alpha_1 + \alpha_2\delta + \alpha_3\gamma t\right), \tag{30}$$

in the case of the series approximation models. The quantities denoted by α_k(k = 1, 2, 3) in Equations (29) and (30) are coefficients determined from the calibration process, whilst $\hat{Y}_n$(n = 2, 3, . . . T) represent total consumption predicted for the nth year. The calibration coefficients are shown in Table 7, for various values of ρ, for the two sets of models.

Table 7. Parameters of QMM utilized for total energy consumption forecasting

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	0.007	0.0464	60.0000	-21.5109
	0.008	0.0845	21.8926	-21.5210
	0.025	0.1528	-47.1875	-21.6893
	0.700	0.0455	33.0269	-23.4053
	0.850	0.1801	-107.0417	-22.8855
Harvey Logistics	2.00	-0.0239	0.9306	N/A
Harvey	1.08	-0.0652	1.3735	N/A

Thus, using the series approximation model defined by $\rho = 0.025$, and the Harvey model as examples, total energy consumption is forecasted according to

$$\hat{Y}_t = \hat{Y}_{t-1} + \hat{Y}_{t-1}^{0.025}\left(0.1528 - 47.1875\delta - 21.6893\gamma t\right), \tag{31}$$

and

$$\hat{Y}_t = \hat{Y}_{t-1} + \hat{Y}_{t-1}^{1.08}\left(e^{(-0.0652\delta + 1.3735\gamma t)}\right), \tag{32}$$

respectively: provided that (δ, γ) =(0.001, 0.0001) in Equation (31) and (50.27, -0.028), in Equation (32). In both equations, t = 2, . . . , 9. Profiles of the total consumption forecasted by the models defined through the parameters of Table 7 and Equation (31) and (32) are displayed in Fig. (4), which also includes the profiles of the corresponding historical fits and actual total energy consumption.

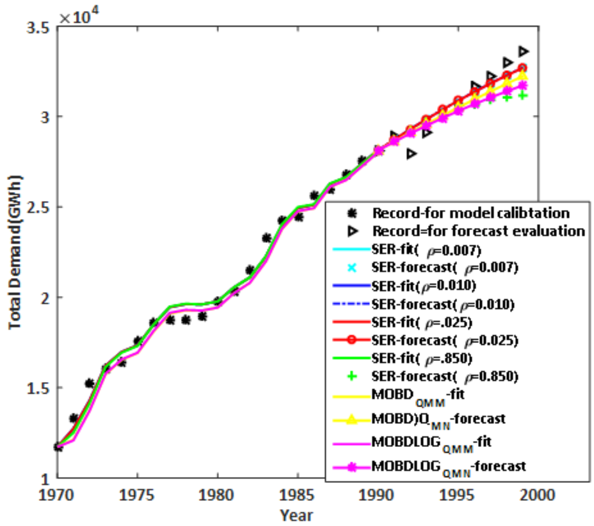

Figure 4. Comparison of energy consumption forecasted by the models defined by Table 7 with actual energy consumption.

Forecasting accuracy, for each of the models is specified by the representative metrics of Table 8, and it is readily observed from both Fig. (4) and Table (8) that all the QMM models very closely track the actual energy consumption for each year, with excellent forecasting accuracies.

Table 8. Parameters of QMM utilized for total energy consumption forecasting

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSPE
Series Approx. ρ = 0.007	-5.30	513.2	1.68	647.18	2.16
Series Approx. ρ = 0.008	27.70	526.9	1.72	663.26	2.20
Series Approx. ρ =0.010	-13.30	510.3	1.67	644.35	2.16
Series Approx. ρ = 0.700	12.20	525.2	2.52	1050.2	3.29

Series Approx. ρ **= 0.850**	422.9	795.3	2.85	1197.5	3.71
Harvey	551.1	903.5	2.00	790.44	2.52
Harvey. Logistic	295.1	625.3	2.53	1008.1	3.15

The metrics (using MAPE as a typical example) indicate that for values of ρ less than 0.70, the forecasting accuracies of the series approximation models are significantly better than those for the Harveyy / Harvey logistic models. However, for higher values of ρ, the Harvey (in particular) and Harvey logistic models perform slightly better than the series approximation models.

3.3. Alternative QMM Models

When instead of the indices (t = 1, 2, , , ,) utilized in the QMM models of the foregoing sections, actual values of year (t = 1944, 1945, . . .) are utilized as in Mohamed and Bodger [18], alternative QMM models become available. For these alternative models the base models of choice are those given by Equations (11) to (16) of [18], for which Equations (21) and (25) of this paper represent modifications that account for the Harvey singularities.

In this section, the prediction performances of the models of [18] are compared with those of their corresponding QMM models, in each case, with t = 1944, 1945, . . . , 1999. Outcomes of the QMM calibration of these alternative models are defined by the parameters of Table 9.

Accordingly, the QMM model derived from the calibration of Equation (24) with non-domestic energy consumption data, for example, emerges as

$$\tilde{Y}_t \simeq \begin{cases} Y_{t-1} - Y_{t-1}^{1.29}\left(1.4387 * 57.46 - 1.4404 * 0.032t\right), & y_t < 0 \\ \text{or} & \\ Y_{t-1} + Y_{t-1} + Y_{t-1}^{1.29}\left(1.4387 * 57.46 - 1.4404 * 0.032t\right), & y_t > 0 \end{cases} \quad t = 1944, \ldots, 1999, \tag{33}$$

Table 9. Calibration Coefficients for the QMM Alternative Models

DOMESTIC CONSUMPTION CASES		
Model/ Parameters	**Harvey**	**Harvey logistics**
α_1	0.7013	0.8323
α_2	0.7018	0.8405
NON-DOMESTIC CONSUMPTION CASES		
α_1	1.4837	1.2096
α_2	1.4404	1.1919
TOTAL CONSUMPTION CASES		
α_1	0.7647	0.8845
α_2	0.7710	0.8856

In addition to the models defined by the calibration coefficients of Table 9, alternative series approximation models were developed through the choices of $(\delta, \gamma) = (0.0001, 0.000001)$, which ensure that for t = 1999, the argument of the exponential function of the Harvey model remains much less than 1, so that the two-term series approximation also remains applicable. The consequent QMM models developed are typified by that which derived from calibration with total energy consumption, and given as

$$\tilde{Y}_t \simeq \begin{cases} Y_{t-1} - Y_{t-1}^{0.85}\left(8.62 + 8404.50\delta - 46825.14\gamma t\right), & y_t < 0 \\ \text{or} & \\ Y_{t-1} + Y_{t-1}^{0.8}\left(8.62 + 8404.50\delta - 46825.14\gamma t\right), & y_t > 0 \end{cases} \quad t = 1944, \ldots, 1999, \tag{34}$$

As before, all numerals with magenta colored fonts in Equations (31)–(34) identify model calibration coefficients.

Figure 5 displays the profiles of energy consumption predicted by various alternative QMM models compared with predictions by corresponding models of [18], as well as the actual energy consumptions, with which the QMM base models were calibrated. The prediction accuracies of these alternative models are described by the performance metrics shown in Table 10 , from which a number of important conclusions are indicated.

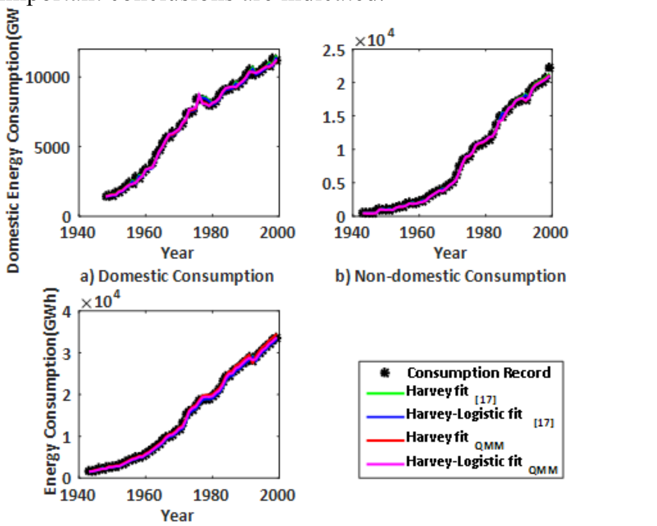

Figure 5. Profiles of energy consumption predicted by the alternative QMM models of Table 9.

Table 10. Prediction Performance Metrics for the Alternative Models Defined by Table 9

DOMESTIC CONSUMPTION CASES					
Model/Metric	**MPE**	**MAE**	**MAPE**	**RMSE**	**RMSPE**
Harvey Mohamed and Bodger [18]	35.92	194.87	3.66	268.58	5.46
Harvey-logMohamed and Bodger(2005)	119.96	215.54	4.03	268.58	5.46

QMM-HarveyMohamed and Bodger [18]	96.29	152.70	3.01	204.49	4.30
QMM-Harvey-logMohamed and Bodger [18]	86.18	156.24	3.01	205.78	4.42
NON-DOMESTIC CONSUMPTION CASES					
HarveyMohamed and Bodger[18]	89.57	285.82	6.49	404.88	10.40
Harvey-logMohamed and Bodger [18]	151.51	288.64	6.57	404.88	10.40
QMM-HarveyMohamed and Bodger [18]	185.11	255.70	5.44	375.67	8.84
QMM-Harvey-logMohamed and Bodger[18]	176.23	256.69	5.39	378.39	8.96
TOTAL CONSUMPTION CASES					
HarveyMohamed and Bodger(2005)	387.23	462.81	4.53	657.31	6.32
Harvey-logMohamed and Bodger[18]	469.28	525.09	5.00	57.31	6.32
QMM- Harvey Mohamed and Bodger [18]	- 187.47	350.96	2.80	474.51	3.70
QMM-Harvey-logMohamed and Bodger [18]	166.31	312.29	3.41	413.17	4.59
SERIES APP.	1.83	247.84	2.38	361.10	3.48

First, the MAPE metrics of Table 10 indicate that the alternative (‘t’ expressed as ‘year’) QMM models perform better in prediction than the base models from which they derive. And for both the base and QMM models, the Harvey models have better MAPE metrics than the Harvey logistic models, though the series approximation model’s MAPE is better than those of either of them. This is also generally true of all the other metrics in the table. Second, a comparison of the metrics of Table 10 with corresponding metrics of Tables 2, 4, and 6 also very clearly reveal that metrics due to the choice of t = 1, 2, 3, . . .,in general, define better performing QMM models than obtains with the use of t = 1944, 1945, ... It is worth noting, nonetheless, that these differences are most likely due to the effects of computational round offs, which can be expected to be significant on account of the larger magnitudes of the variable ‘t’ in the alternative models.

3.4. Further QMM Performance Evaluation

Towards providing further validation for the QMM model calibration process as a tool for the prediction of energy demand (consumption), two additional examples from the literature, Okakwu et al [15], Ogungbemi et al [16], are considered in this section. In both examples, outcomes of the QMM calibration of base Harvey and Autoregressive models are compared with corresponding results reported in [15] and [16].

3.4.1 Calibration with Annual Peak and Average Demand Data from [15]

Parameters of the QMM models due to the calibration of the base models of Equations (14) and (15), (for the Harvey and series approximation models, respectively) with annual peak demand data, are displayed in Table 11.

Table 11. Parameters of QMM models utilized for annual peak energy demand prediction

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	0.007	22.16	124880.87	22632.74
	0.010	343.65	-200000.00	21904.68
	0.015	-25.72	- 163909.11	20739.82
	0.020	419.63	-286704.12	19633.33
	0.025	162.83	-34965.27	18582.61
Harvey Logistics	2.00	0.03	2.17	N/A
Harvey	0.010	-0.08	-1.41	N/A

Corresponding profiles of peak annual energy demand predicted by the models are presented by Fig. 6, which includes the energy prediction curves due to autoregressive-based QMM models. The QMM-autoregressive models derive from Equation (9), with base model parameters given as $(\beta_0, \sigma) = (1.05,\ 0.083)$, and calibration coefficients obtained as $(\alpha_1, \alpha_2) = (163.3811,\ 0.0347)$.

Prediction accuracies for the models are specified by the performance metrics of Table 12 from which is readily observed that the predictions by the QMM models very closely track the actual peak demand and are comparable, as suggested by Fig. (6).

These performance metrics should ordinarily be compared with corresponding metrics (for the Harvey and autoregressive models) reported in [15]. However, because [15] did not provide parameters of the models (which should have represented the main basis for any claims in the paper) it is not possible to verify the validity of the results reported (Table 3 and Figure 2 of [15] in this particular case) in the paper. As a matter of fact, the metrics reported in Table 2 of [15] for MAPE and MAE are easily shown to be either misprints or due to typographical errors.

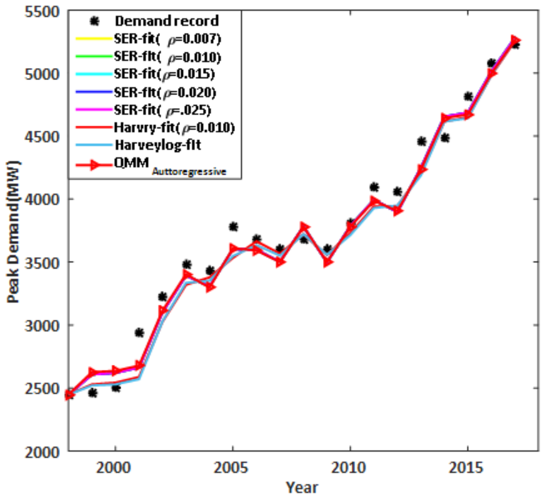

Figure 6. Peak annual energy demand predicted by the QMM models, compared with actual annual peak demand of Okakwu et al [15].

Table 12. Prediction performance metrics for the QMM models defined by Table 11

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSPE
Series Approx. ρ = 0.007	56.30	115.95	3.26	130.94	3.83
Series Approx. ρ = 0.010	56.31	115.96	3.26	0.95	3.83
Series Approx. ρ =0.015	56.34	115.97	3.26	130.96	3.83
Series Approx. ρ = 0.020	56.36	115.99	3.26	130.98	3.83
Series Approx. ρ = 0.025	56.39	116.00	3.26	130.99	3.83
Harvey	80.65	111.86	3.06	143.51	4.09
Harvey. Logistic	86.27	116.41	3.16	147.10	4.18
Autoregressive	56.87	116.94	3.30	131.55	3.87

The demand forecasting capabilities of the QMM models were further evaluated with the calibration of the base models with peak demand data covering 2002 to 2012, and using the resulting QMM models to forecast peak demand for the years extending from 2013 to 2017. An example of QMM demand forecasting models is given below as

$$\hat{Y}_t = \hat{Y}_{t-1} + \hat{Y}_{t-1}^{0.01}\left(243.02 - 123777.47\delta + 59443.67\gamma t\right), \quad (34)$$

a series approximation model for which $\rho = 0.010$, with model parameters $(\delta,\gamma) = (0.001, 0.0001)$, and whose associated model calibration coefficients are displayed in magenta colored fonts. The curves displayed in Fig. (7) describe the forecasting performances of the models, and the metrics of Table 13 quantify their forecasting capabilities. It is clear from the curves of Fig. (7) that the forecasts by the series approximation models are virtually the same for all values of ρ, and are significantly better than the forecast due to the Harvey model. This observation is solidly supported by the performance metrics of Table 13.

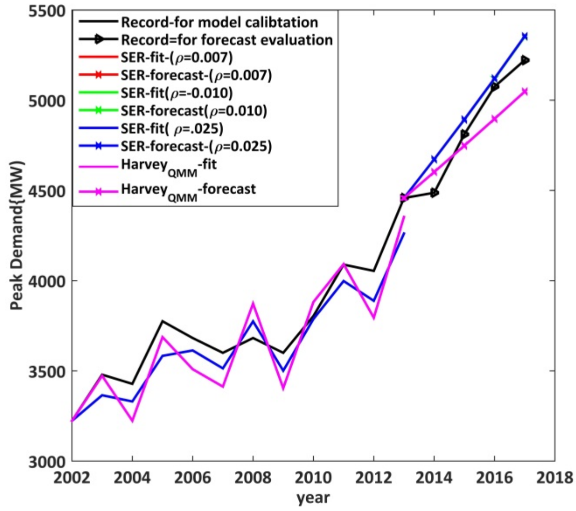

Figure 7. Comparison of annual peak demands forecasted (2013-2017) by QMM models with actual peak demand

Table 13. Forecasting performance metrics for the QMM models described by Fig. (7)

METRIC/ MODEL	MPE	MAE	MAPE	RMSE
Series Approx. ρ = 0.007	87.8060	87.8060	1.8286	2.3103
Series Approx. ρ = 0.010	-88.0340	88.0340	1.8331	2.3143
Series Approx. ρ = 0.025	-89.1320	89.1320	1.8549	2.3331
Harvey. ρ = 0.010	60.0880	106.0480	2.1432	2.5192

Using the same base autoregressive and other models defined by the values of (as well as corresponding values of) in Table 11, QMM prediction models were developed through calibration with the annual average demand data of Okakwu et al [15]. Parameters of the QMM models are as presented in Table 14.

Table 14. Parameters of QMM models utilized for annual average energy demand prediction

Model/ Parameter	ρ	α_1	α_2	α_3
	0.007	-55.84	185022.45	45626.06
	0.010	297.64	-171463.52	44415.96

Series Approx.	0.015	57.07	64267.13	42469.19
	0.020	92.73	23945.31	40607.37
	0.025	-75.00	187194.54	38826.67
Harvey Logistics	2.00	-0.07456	-1.8291	N/A
Harvey	0.010	0.0277	1.6736	N/A

Annual average demands predicted by these models are described by the profiles of Fig. (8), which also include the distribution of actual annual average demand. And The metrics of Table 15 represent the quantitative measures of the accuracies of the predictions due to the profiles of Fig. (8).

In general, the annual average demand prediction performances of the models are similar to those described for annual peak demand prediction. An important difference between the two is typified by the MAPE metrics of Table15, which indicates that unlike what obtained with peak demand prediction, the autoregressive model is better performing than the Harvey models.

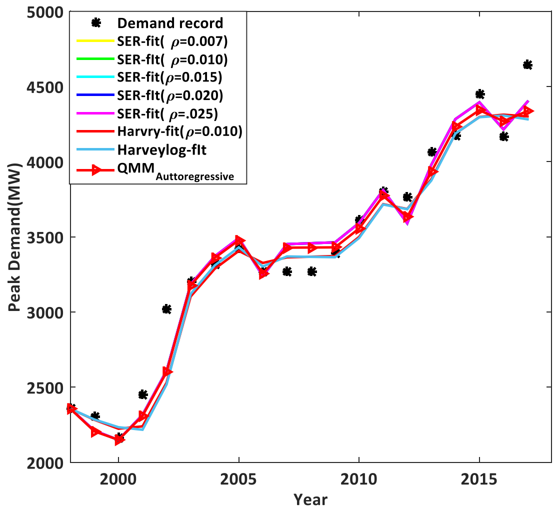

Figure 8. Comparison of actual average demand with annual average demands forecasted by QMM models.

Table 15. Prediction performance metrics for the QMM models defined by Table 14

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSPE
Series Approx. ρ = 0.007	25.37	99.57	2.98	139.71	4.29
Series Approx. ρ = 0.010	25.38	99.56	2.98	139.72	4.29
Series Approx. ρ =0.015	*25.40*	99.56	2.98	139.74	4.30
Series Approx. ρ = 0.020	25.42	99.56	2.98	139.76	4.30
Series Approx. ρ = 0.025	25.43	99.55	2.98	139.78	4.30
Harvey	66.53	114.46	3.36	163.68	4.95
Harvey. Logistic	68.03	115.68	3.41	169.13	5.13
Autoregressive	40.82	103.49	3.04	144.44	4.35

3.4.2. Calibration with Annual Industrial Sector Consumption Data from [16]

Annual energy demand by the industrial sector in Nigeria, [16] is, unlike those in [15] and [18], characterized by quite a few instances of very sharply alternating variations in demand recorded for consecutive years. In this section therefore, the responses of the base models of the previous sections to QMM calibration are investigated for possible additional information about the algorithm's properties.

QMM models developed through the calibration of various base models are defined by the parameters of Table 16.

Table 16. Parameters of QMM models utilized for annual industrial energy demand prediction

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	1.08	0.0086	85.7829	2.8752
	1.32	0.0124	14.1483	0.8847
	185	0.0011	0.4184 -	-0.1965
	2.00	-0.0019	2.5406	-0.0996
Harvey Logistics	2.00	-0.3034	-0.2226	N/A
Harvey	1.32	-0.1481	0.0930	N/A
Autoregressive	N/A	2.5723	0.0059	N/A

And the profiles of predictions by the models are displayed in Fig. (9).

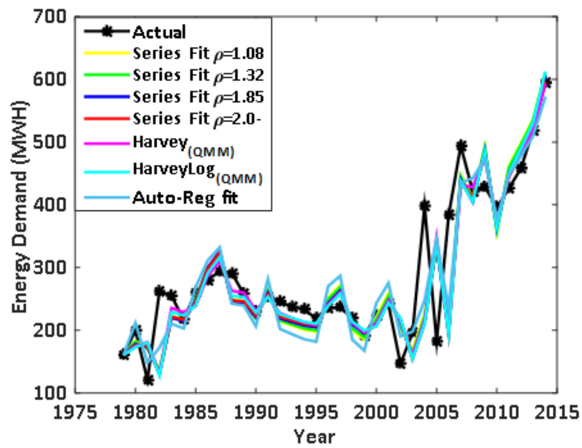

Figure 9. Profiles of annual industrial energy demand forecasted by the QMM models defined by Table 16

These curves suggest that the series approximation-based models have virtually the same prediction profiles, which differ slightly from those due to the Harvey / Harvey logistic and autoregressive models.

A clearer picture of the relative performances of the models is offered by the prediction accuracy metrics of Table 17. According to the metrics, MPE and RMSE, for the models follow the same general pattern in that for the series approximation models, the metrics improve with decreasing values of ρ with the exception of the metrics of MPE and RMSE for the Harvey model, which are better than those recorded for the series approximation models, with the autoregressive model recording the best metrics.

Table 17. Performance metrics for the QMM models defined by Table 16.

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSPE
Series Approx. ρ = 1.08	8.45	39.30	14.79	58.98	22.72
Series Approx. ρ = 1.32	9.27	39.38	14.84	59.89	23.03
Series Approx. ρ = 1.85	10.94	39.03	14.86	61.50	23.74
Series Approx. ρ = 2.00	11.62	37.47	14.52	62.01	24.21
Harvey	9.18	35.24	13.81	60.78	24.35
Harvey. Logistic	9.96	37.83	14.47	62.15	24.35
Autoregressive	7.77	41.02	15.65	56.15	22.31

The MAE metrics for the series approximation models also display the same trend, except that MAE for (incidentally about the same as that for the Harvey logistic model) is the best among them. The Harvey model recorded the best MAE, and the autoregressive model, the worst. The QMM-Harvey model recorded the best value of MAPE, and the QMM-autoregressive model, the worst. On the other hand, RMSPE recorded for the autoregressive model is slightly better than for all the other models, with those for the series approximation models being better than the metrics for the Harvey / Harvey logistic models.

The foregoing observations compared with similar assessments in the previous sections underscore the remark in [1] that whereas MAPE (on account of its simplicity and transparency) is generally regarded as the best metric for prediction performance comparisons, the suitability of a model for any scenario depends significantly on the dataset. Indeed, the fact that MAPE values recorded in this case are all greater than the Lewis benchmark of 10% [1] for high prediction accuracy does not suggest poor prediction accuracy: but may be indicative of excellent performance for the dataset of this variety.

Again, the QMM models' performances should ordinarily be compared with corresponding results in [16]. Unfortunately, the paper, which like [15]. appear not to have enjoyed the benefits of a competent peer review process, includes a number of inaccuracies, which seriously question the veracity of its claims. First, [15]'s Equation (5), which incorrectly equates $\log_e\left(Y_t - Y_{t-1}\right)$ to $\log_e\left(\frac{Y_t}{Y_{t-1}}\right)$ does not represent a Harvey model. And second, MATLAB implementations of [15]'s Equation (31) (for the purported 'Harvey model') and Equation (32) (for the autoregressive model) yielded RMSE values far in excess of those reported in the paper.

Alternative models of the types described in section 3.3 (with t = 1979, 1980, in place of t = 1, 2,) were also developed with the industrial energy demand data. Model parameters for the alternative models are displayed in Table 18, as well as for the series approximation models, and for the Harvey models.

Table 18. Parameters of alternative QMM models utilized for annual industrial energy demand prediction

Model/ Parameter	ρ	α_1	α_2	α_3
Series Approx.	1.08	-0.1613	-3101.6	286.09
	1.32	-0.05	2499.50	-89.23
	185	0.0633	-232.3816	-19.4829
	2.00	0.0209	-5.0000	-9.9826
Harvey Logistics	2.00	0.5678	-0.2202	N/A
Harvey	1.32	-0.4115	0.0629	N/A
Autoregressive	N/A	2.5723	0.0059	N/A

Profiles of demand predicted by the alternative models are displayed in Fig. (10). A cursory comparison of Fig. (9) and (10) will suggest that the models developed using t = 1, 2, . . . are, in this case, equivalent in performance with those developed with the use, in this case, of t = 1978, 1979, ...

This, as earlier remarked, is not surprising, because the QMM algorithm, being a calibration process, can be expected to adjust the model calibration coefficients to suit whichever form of 't' may be preferred.

Because the metrics of Table 19 very closely match the corresponding metrics of Table 17, discussions and associated conclusions concerning the latter also apply in this case. The differences between the two sets of metrics of this section are smaller than those of section 3.3 because the effects of computational round off are more pronounced in the latter, on account of the relatively large magnitudes of data involved.

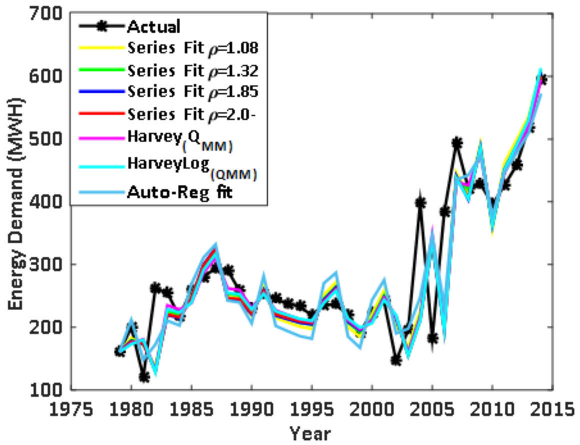

Figure 10. Profiles of annual industrial energy demand forecasted by the alternative QMM models defined by Table 18.

Table 19. Performance metrics for the QMM models defined by Table 18

METRIC/ MODEL	MPE	MAE	MAPE	RMSE	RMSPE
Series Approx. $\rho = 1.08$	8.46	39.29	14.79	58.98	22.72
Series Approx. $\rho = 1.32$	9.69	36.73	14.18	60.18	23.62
Series Approx. $\rho = 1.85$	10.88	39.24	14.91	61.50	23.70
Series Approx. $\rho = 2.00$	11.51	38.12	14.68	61.93	24.07
Harvey	9.27	35.30	13.83	60.91	24.36
Harvey. Logistic	9.94	37.92	14.49	62.14	24.33
Autoregressive	7.77	41.02	15.65	56.15	22.31

4. CONCLUSION

This paper, using the Harvey, Harvey logistic, and Autoregressive models as candidate 'base models', has introduced the Quasi-Moment-Method (QMM) as a novel approach to the development of high performance models for the prediction of electric energy demand.

For the purposes of formulation validation as well as performance evaluation and in order to facilitate easy verification, energy demand / consumption data from three different publications were utilized for the calibration of the base models, to develop corresponding QMM models. And as may be expected, the computational results revealed that the QMM models generally performed better than the corresponding classical (nominal) Harvey and autoregressive models. A number of interesting attributes were revealed by the results. First, the performance metrics due to the series approximation QMM models were generally better than the corresponding metrics for Harvey models, evidently on account of computational round-offs associated with exponential and logarithmic functions. Second, when the time variable denoted by 't' is taken as 1, 2, 3, . . . , (as against 1940, 1941, 1942 . . . , for example) significantly improved performance metrics are in general, recorded, again, on account of the computational round-off involved with the use of larger numbers. And third, the results confirm that RMSE represents a better model performance metric than MAPE or RMSPE, because these latter relative error measures tend to suggest that the prediction errors due to different models are much closer than they actually are. It should be remarked nonetheless, that it is probable that when the inevitable influence of computational round off is discounted, the QMM models may be considered as being essentially equivalent: or put in other words, that when the components of the base model satisfy the linearity requirement, the QMM solution is unique.

Future investigations should examine the uniqueness property indicated by the results of this paper for the QMM solution, as well as explore the possibility of hybridization with machine learning algorithms, particularly the metaheuristic methods.

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