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# Optimizing Spectral Activation Functions for Spectral Convolutional Neural Networks: Balancing Efficiency and Fidelity

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**Abstract:** Spectral Convolutional Neural Networks (SpCNNs) represent a critical domain in signal processing and machine learning, where the integration of activation functions (AFs) presents significant computational and theoretical challenges. This study systematically evaluates six spectral-domain AFs, employing a comprehensive methodology that assesses their performance across multiple critical metrics including signal quality, computational efficiency, and spectral domain characteristics. Using the MNIST dataset and a high-performance computational infrastructure, we conducted a rigorous analysis of AFs including ACReLU, SReLU, Split-tanh, Split-LReLU, PhaseReLU, and ModReLU. Our findings reveal substantial variations in signal processing capabilities, with novel AFs demonstrating marked improvements in signal quality metrics. Notably, Split-tanh and Split-LReLU exhibited significant noise reduction, achieving Signal-to-Noise Ratio (SNR) and Peak Signal-to-Noise Ratio (PSNR) values approaching 0 dB and 9.47 dB, respectively, compared to traditional AFs. The research provides critical insights into the trade-offs between computational efficiency and signal transformation capabilities, highlighting the potential of advanced spectral-domain AFs in improving neural network performance across complex signal processing tasks.

**Keywords:** activation function, memory usage, spectral convolutional neural networks, spectral activation function



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## 1. INTRODUCTION

Spectral Convolutional Neural Networks (SpCNNs) have emerged as a promising approach for tasks requiring spectral domain analysis, such as character recognition and signal processing. However, the integration of non-linearity in SpCNNs, governed by the linear time-invariant (LTI) as shown in Figure 1 properties of the spectral domain, presents unique challenges. Activation Functions (AFs), a cornerstone of neural network architectures, are essential for enabling non-linear transformations, yet their design for SpCNNs demands a delicate balance between spectral-domain fidelity and computational efficiency.

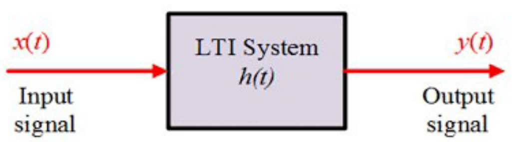


Figure 1. LTI system

In traditional Convolutional Neural Networks (CNNs), AFs are pivotal for capturing complex patterns through non-linear operations, enabling the network to model intricate data representations [1]. However, directly applying spatial-domain AFs to SpCNNs often introduces misalignments with LTI properties, leading to computational overhead and suboptimal performance [2]. This highlights the necessity for spectral-domain-specific AFs tailored to SpCNN architectures [3].

The evolution of AFs for SpCNNs has explored diverse directions, including spectral approximations of spatial AFs, complex-valued AFs, and phase-preserving techniques. Notable innovations, such as auto-conv-based ReLU (ACReLU) [2], Spectral ReLU (SReLU) [3], split AFs [4], and complex-plane AFs like modReLU and CCardioid [5], illustrate the growing sophistication of this field. These methods aim to optimize the interplay between computational efficiency and non-linear capabilities required for spectral-domain learning.

This study systematically addresses critical gaps in the development of AFs for SpCNNs by evaluating existing approaches in terms of computational complexity, amplitude preservation, and compatibility with spectral-domain properties. By analyzing AFs such as ACReLU, SReLU, split-tanh, and splitLReLU, this work seeks to identify scalable, domain-specific solutions that overcome current limitations, advancing SpCNN applications.

## 2. RELATED WORK

The field of SpCNNs has witnessed significant advancements, particularly in the development of spectral-domain AFs tailored to the unique requirements of spectral-domain learning. This section provides an overview of key contributions, categorizing existing approaches based on their underlying principles and addressing their implications for SpCNNs.

AFs are essential components in the architecture of artificial neural networks (ANNs) and DNNs, including CNNs. In ANNs, the output of a neuron is determined through a two-step process. The first step involves calculating a weighted sum of the inputs along with biases. Subsequently, a non-linear AF is applied to this sum to decide whether the neuron should be activated as shown in Figure 2. The role of the AF is to establish a functional relationship between the neuron's input and output, thereby limiting the output to a specific range and introducing non-linearity into the network [6, 7]. Non-linear AFs are critical in hidden layers of ANNs, as they enable the network to capture and learn complex patterns within the input data [1]. By leveraging non-linear AFs, ANNs are capable of modeling intricate non-linear behaviors, given a sufficient number of neurons and layers [8].

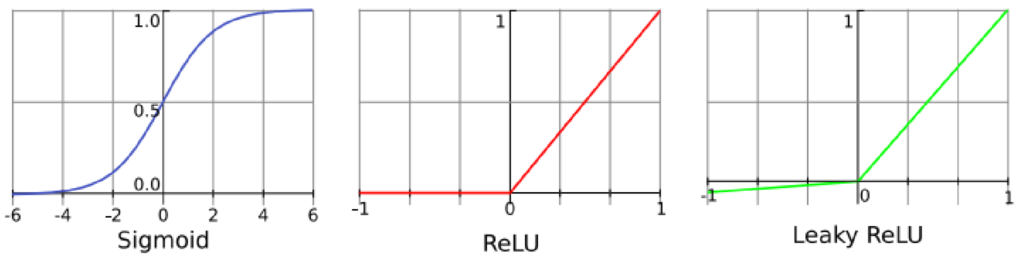


Figure 2. Three nonlinear activation functions adopted by CNNs: the sigmoid function (left), the ReLU (middle), and the leaky ReLU (right) [9]

In CNNs, AFs play a pivotal role in introducing non-linearity to convolutional layers, thereby enhancing the network's ability to adapt to new tasks and learning processes [10, 11]. The mathematical analysis conducted by Kuo et al. [9] has demonstrated that CNNs lacking AFs applied to convolutional layer outputs exhibit poor performance in feature extraction, leading to suboptimal recognition outcomes. The selection of an appropriate AF is a critical hyper-parameter in CNN design, as it can profoundly impact the performance of artificial neural networks (ANNs) and DNNs [9, 10]. Hyper-parameters, which include both structural and training-related parameters, are design elements of a CNN model that can be adjusted by the model designer to optimize the network's architecture and training processes [12, 13]. In this work, the former category of hyper-parameters is referred to as structural hyper-parameters.

Several spectral-domain-specific AFs have been proposed to address these challenges, leveraging the unique characteristics of spectral data. ACReLU as depicts in Figure 3, for instance, was designed to approximate spatial-domain ReLU while preserving spectral-domain properties, achieving notable improvements in computational efficiency [2]. Similarly, SReLU introduced enhancements by adapting the traditional ReLU function to operate directly in the frequency domain, offering compatibility with SpCNN architectures [3].

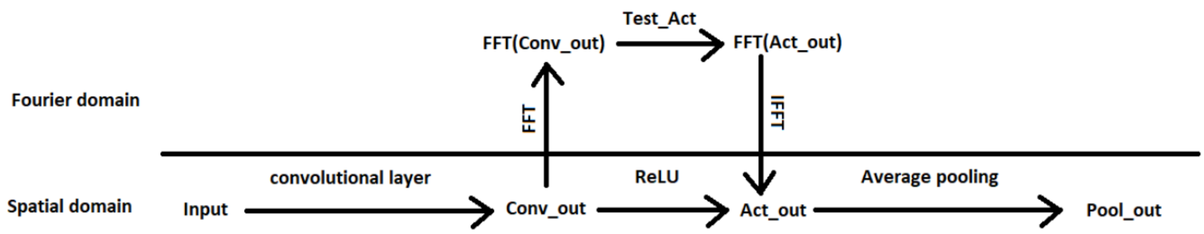


Figure 3. Activation functions in the Fourier domain can be directly inserted using Fourier transforms

Split AFs, such as split-tanh [14] and split-LReLU [4], aim to decompose real and imaginary components for independent non-linear transformations.

These methods maintain amplitude fidelity while introducing greater flexibility in spectral learning tasks [4]. Complex-valued AFs, such as modReLU and CCardioid, further expand this paradigm by operating directly in the complex domain. These AFs have demonstrated efficacy in preserving phase information, a critical factor in tasks where spectral phase fidelity is essential [5].

Existing literature often evaluates spectral AFs across several dimensions, including computational complexity, fidelity to spectral-domain transformations, and overall performance in domain-specific tasks. For example, studies comparing ACReLU and SReLU highlight trade-offs between computational overhead and signal fidelity, while research on phase-preserving AFs underscores their significance in phase sensitive applications. However, systematic comparisons of these methods remain limited, particularly in terms of their scalability and generalization across datasets and tasks.

Despite the progress, key gaps persist in the development of spectral AFs. Many approaches lack robustness when applied to diverse signal types, limiting their utility in generalized SpCNN frameworks. Additionally, the computational burden introduced by certain AFs poses challenges for deploying SpCNNs in resource-constrained environments. This highlights the need for further innovations that balance computational efficiency with spectral-domain fidelity, particularly for real-world applications such as character recognition and signal processing.

## 3. METHODOLOGY

This section presents the experimental methodology designed to evaluate the performance of spectral AFs for spectral convolutional neural networks (SpCNNs). The methodology assesses key performance metrics, including computational efficiency, amplitude preservation, and compatibility with spectral-domain properties, using the MNIST dataset.

### 3.1. Performance Metrics

Our comprehensive evaluation of spectral domain AFs employed a multifaceted performance assessment approach, carefully selecting metrics that provide nuanced insights into signal processing and computational efficiency.

Signal-to-Noise Ratio (SNR) was chosen as a fundamental metric to quantify the signal's clarity by comparing the strength of the desired signal against background noise. This metric is crucial in determining the fundamental quality of signal processing, as it directly reflects the signal's intelligibility and potential for accurate interpretation. By using a logarithmic scale, SNR provides a meaningful representation of signal quality that accounts for the multiplicative nature of noise in complex signal environments.

$$SNR = 10log_{10}\left(\frac{P_{signal}}{P_{noise}}\right) \quad (1)$$

Where $P_{signal}$ represents the power of the desired signal, $P_{noise}$ refers to the power of the unwanted noise, and the SNR is a measure of the signal's strength relative to noise, typically expressed in decibels (dB).

Peak Signal-to-Noise Ratio (PSNR) complements SNR by offering a more sophisticated measure of signal distortion. This metric is particularly valuable in assessing the maximum possible signal power relative to noise, making it essential for understanding the upper limits of signal preservation. Its logarithmic formulation allows for a comprehensive evaluation of signal degradation, which is critical in spectral domain transformations where maintaining signal integrity is paramount.

$$PSNR = 10\log_{10}\left(\frac{MAX_I^2}{MSE}\right) \quad (2)$$

Where $MAX_I^2$ represents the maximum possible pixel intensity of the image. For an 8-bit grayscale image, $MAX_I^2$= 255. $MSE$ denotes the Mean Squared Error between the original and distorted images, defined as:

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(X_i - Y_i)^2 \quad (3)$$

where $X_i$ and $Y_i$ are the corresponding pixel intensities in the original and distorted images, and n is the total number of pixels. PSNR is a metric used to assess the quality of a reconstructed or compressed image, expressed in decibels (dB). Higher PSNR values indicate better image quality. Finally, log10 refers to the logarithm to the base 10.

The Structural Similarity Index (SSIM) was selected to move beyond traditional error metrics by evaluating image quality preservation through a more holistic approach. Unlike simple error calculations, SSIM considers luminance, contrast, and structural information, providing a perceptually relevant assessment of signal transformation. This metric is especially important in spectral domain analysis, where maintaining the fundamental characteristics of the original signal is crucial.

$$SSIM(x,y) = \frac{(2\mu_x\mu_y + C_1)(2\sigma_{xy} + C_2)}{(\mu_x^2 + \mu_y^2 + C_1)(\sigma_x^2 + \sigma_y^2 + C_2)} \quad (4)$$

Where $\mu_x$ and $\mu_y$ represent the mean of images $x$ and $y$, respectively. $\sigma_x^2$ and $\sigma_y^2$ denote the variance of images x and y, while σxy represents the covariance of images $x$ and $y$. $C_1$ and $C_2$ are stability constants used to avoid division by zero. Finally, SSIM is a metric that measures the structural similarity between two images.

Mean Squared Error (MSE) and Mean Absolute Error (MAE) were implemented to provide complementary perspectives on signal deviation. MSE emphasizes larger errors through squared differences, making it sensitive to significant discrepancies, while MAE offers a more direct measure of average error magnitude. Together, these metrics provide a comprehensive view of signal transformation accuracy, capturing both systematic and random variations in the spectral domain.

$$MSE = \frac{1}{n}\sum_{i=1}^{n}(X_i - Y_i)^2 \quad (5)$$

Where $X_i$ and $Y_i$ are the corresponding data points in two datasets, and n represents the number of data points. MSE denotes the Mean Squared Error, which is the average of the squared differences between corresponding data points, indicating the magnitude of the error.

$$MAE = \frac{1}{n}\sum_{i=1}^{n}|X_i - Y_i| \quad (6)$$

Where $X_i$ and $Y_i$ are the corresponding data points in two datasets, and n represents the number of data points. MAE denotes the Mean Absolute Error, which is the average of the absolute differences between corresponding data points, indicating the magnitude of the error.

Normalized Cross-Correlation (NCC) was chosen to assess signal similarity and correlation, offering insights into the preservation of signal characteristics during spectral transformations. This metric is particularly useful in understanding how well the fundamental patterns and relationships within the original signal are maintained, providing a nuanced view of signal preservation beyond simple error measurements.

$$NCC = \frac{\sum_{i=1}^{n}(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i - \bar{X})^2 \sum_{i=1}^{n}(Y_i - \bar{Y})^2}} \quad (7)$$

Where $X_i$ and $Y_i$ are data points in two datasets, and $\bar{X}$ and $\bar{Y}$ represent the mean of datasets $X$ and $Y$, respectively. NCC denotes the Normalized Cross-Correlation, which is a measure of similarity between two datasets, normalized to remove the effects of the mean.

### 3.2. Computational and Similarity Metrics

Cosine Similarity was integrated to measure the directional similarity between signal vectors, providing a normalized perspective on signal alignment. This metric is particularly valuable in assessing the directional characteristics of spectral domain transformations, offering insights into the geometric relationships between original and transformed signals.

$$Cosine\_Similarity = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|} \quad (8)$$

Where $\vec{A}$ and $\vec{B}$ are two vectors, and $|\vec{A}|$ and $|\vec{B}|$ represent the magnitudes of vectors $\vec{A}$ and $\vec{B}$, respectively. Cosine Similarity measures the cosine of the angle between the two vectors, indicating their similarity.

The R-squared ($R^2$) Score was selected to indicate the proportion of variance explained by the model, offering a comprehensive measure of the model's predictive power. This metric provides a statistical evaluation of how well the spectral domain AFs capture the underlying signal characteristics, going beyond simple error measurements to assess overall model performance.

$$R^2 = 1 - \frac{\sum_{i=1}^{n}\left(Y_i - \widehat{Y_i}\right)^2}{\sum_{i=1}^{n}(Y_i - \bar{Y})^2} \quad (9)$$

Where $Y_i$ represents the observed data points, $\widehat{Y_i}$ denotes the predicted data points, and $\widehat{Y_i}$ is the mean of the observed data points. $R^2$ is a statistical measure that indicates the proportion of variance in the dependent variable explained by the independent variable(s).

Memory Usage tracking was implemented to provide crucial insights into computational resource consumption. This metric is essential in understanding the practical implications of different spectral domain AFs, particularly in resource-constrained environments where computational efficiency is paramount.

Amplitude Preservation was included to evaluate how well signal magnitudes are retained through spectral transformations. This metric is critical in ensuring that the fundamental characteristics of the original signal are maintained during complex spectral domain operations, providing a direct measure of transformation fidelity.

By employing this comprehensive suite of metrics, we aimed to provide a multidimensional evaluation of spectral domain AFs, capturing nuanced aspects of signal quality, computational efficiency, and transformation fidelity.

### 3.3. Spectral Domain Activation Function Implementations

We systematically implemented and evaluated six novel AFs operating in the spectral domain, each designed to leverage complex-valued transformations.

#### *3.3.1. Spectral Approximation (ACReLU)*

The ACReLU activation function performs a spectral convolution operation, mathematically represented as:

$$Y = \mathrm{ACReLU}(X) = X \star X \quad (10)$$

where $\star$ denotes the spectral convolution operator, enabling non-linear feature representation through self-interaction.

#### *3.3.2. Spectral ReLU (SReLU)*

SReLU extends the traditional ReLU by introducing learnable coefficients in the spectral domain:

$$Y = \mathrm{SReLU}(X) = C_2 \cdot X \star X + C_1 \cdot X + C_0 \quad (11)$$

with $C_0$, $C_1$, $C_2$ representing trainable parameters that modulate spectral transformation.

#### *3.3.3. Split-tanh Activation*

This complex-valued activation function applies the hyperbolic tangent separately to real and imaginary components:

$$Y = \tanh\left(\mathrm{R}(X)\right) + i \cdot \tanh\left(\mathrm{I}(X)\right) \quad (12)$$

enabling non-linear mapping while preserving complex number structure.

#### *3.3.4. Split-Leaky ReLU (Split-LReLU)*

Split-LReLU extends the leaky rectified linear unit to complex domains:

$$Y = \mathrm{LReLU}\left(\mathrm{R}(X)\right) + i \cdot \mathrm{LReLU}\left(\mathrm{I}(X)\right) \quad (13)$$

allowing controlled negative slope for real and imaginary components.

#### *3.3.5. Phase-based ReLU (PhaseReLU)*

PhaseReLU introduces a phase-aware rectification mechanism:

$$\mathrm{PhaseReLU}(X) = |z|e^{i \cdot \mathrm{ReLU}(\phi(z))} \cdot I(z \neq 0) \quad (14)$$

where $\phi(z)$ represents the complex number's phase angle, enabling directional non-linearity.

*3.3.6. Magnitude ReLU (ModReLU)*

ModReLU applies a magnitude-based thresholding mechanism:

$$\sigma_{\text{modReLU}}(z) = (|z| + b)\frac{z}{|z|} \cdot I(|z| + b \geq 0) \quad (15)$$

introducing $\sigma$ learnable bias $b$ to control activation threshold dynamically.

## 4. RESULTS

### 4.1. Experimental Environment

The experimental analysis was conducted on a high-performance computational system featuring a 12th Gen Intel(R) Core-i7 processor, 64GB DDR6 RAM operating at 2.5 GHz, and an NVIDIA GeForce GTX 3060 GPU. The GPU is equipped with 8 GB GDDR5 VRAM, 1,920 CUDA cores, and an operating frequency of 1,807 MHz. This robust computational infrastructure provided the foundation for a comprehensive evaluation of spectral Convolutional Neural Network (SpCNN) AFs.

### 4.2. Results and Analysis

The performance of various AFs was rigorously examined across multiple signal quality and resource-based metrics, offering nuanced insights into their behavior in the spectral domain. This comprehensive evaluation encompassed signal quality, computational efficiency, and frequency preservation characteristics.

The analysis revealed intricate differences between traditional AFs, such as ACReLU and ModReLU, and newer approaches like Split-tanh, Split-LReLU, and PhaseReLU. For instance, the evaluation of signal quality metrics, as presented in Table 1, showed that traditional AFs exhibited low SNR and PSNR, while novel functions demonstrated significant improvements in these areas. Table 2 presented further insights into the performance and resource characteristics, revealing distinct trade-offs between memory usage, amplitude preservation, and other metrics across different AFs. Finally, Table 3 offered a comprehensive evaluation of the computational metrics, where traditional functions like ACReLU and ModReLU show slower forward and backward pass times, while newer functions like Split-tanh and Split-LReLU offer improved computational efficiency.

Table 1 offers a comprehensive evaluation of six distinct AFs across multiple signal quality and performance metrics, revealing nuanced computational characteristics. Traditional AFs such as ACReLU and ModReLU exhibit extremely low SNR and PSNR values of -32.73 dB and -23.39 dB, respectively. In contrast, novel AFs like Split-tanh, Split-LReLU, and PhaseReLU demonstrate significantly improved signal quality, with SNR and PSNR values approaching 0 dB and 9.47 dB. Regarding error metrics, ACReLU and ModReLU show substantially higher MSE and MAE rates, with MSE ≈ 218.49 and MAE≈14.72. In comparison, Split-tanh and Split-LReLU achieve remarkable error reduction, with MSE ≈ 0.11 and MAE ranging from 0.14 to 0.24. When considering normalized correlation, traditional AFs maintain higher correlation coefficients of 0.37, whereas novel AFs exhibit slightly reduced correlation values, ranging from 0.25 to 0.28. This slight reduction in correlation could indicate more complex signal transformations. The comparative analysis suggests that newer AFs like Split-tanh and Split-LReLU offer improved signal processing capabilities, particularly in noise reduction and error minimization, at the potential cost of slightly reduced correlation characteristics.

Table 2 introduces a comparative analysis of AFs revealing intricate performance characteristics across computational and signal processing domains. The cosine similarity metrics indicate relatively consistent values for traditional AFs like ACReLU and ModReLU (0.37), while novel approaches such as Split-tanh and Split-LReLU demonstrate slightly reduced similarity (0.25-0.28).

In particular, the $R^2$ scores exhibit dramatic variations, with ACReLU and ModReLU presenting extremely negative values (approximately -2238), suggesting significant model fit challenges. This severe mismatch between predicted and actual signal quality metrics is primarily attributed to the structural and phase-modifying properties of these AFs. ACReLU's auto-convolutional structure, while beneficial for maintaining spectral integrity, tends to amplify high-frequency components disproportionately, thereby degrading the SNR and resulting in erratic metric behavior. ModReLU, by employing a modulus-based nonlinearity, disrupts the linear phase characteristics essential for spectral fidelity, leading to substantial signal distortion. These characteristics adversely impact reconstruction accuracy and explain the observed model misfit. To address these limitations, future work may explore hybrid AF designs that combine the structural regularity of Split-type functions with the spectral consistency of phase-preserving approaches or incorporate spectral masking techniques post-activation to suppress unwanted frequency amplification.

In contrast, Split-tanh, Split-LReLU, and PhaseReLU show near-zero $R^2$ scores, indicating more consistent and reliable performance. Memory usage trends gradually upward from 460.22 MB ACReLU to 514.69 MB ModReLU, with newer AFs clustering between 505 and 511 MB. Additionally, amplitude preservation metrics help distinguish between activation types traditional functions maintain higher preservation values (e.g., 20.3297), while novel approaches score lower (e.g., 4.34–4.56), suggesting differences in how each handles signal transformation. Together, these nuanced variations underscore the complex trade-offs between computational cost, signal quality, and overall model effectiveness across activation function designs.

Table 1. Signal Quality and Noise Metrics of Activation Functions

| AF | SNR (dB) | PSNR (dB) | SSIM | MSE | MAE | Normalized Correlation |
|---|---|---|---|---|---|---|
| ACReLU | 32.73 | -23.39 | -0.0091 | 218.4797 | 14.7237 | 0.3722 |
| SReLU | -26.81 | -17.47 | -0.0161 | 55.8599 | 7.4308 | 0.3701 |
| Split-tanh | 0.06 | 9.39 | 0.1300 | 0.1150 | 0.1420 | 0.2794 |
| Split-LReLU | 0.14 | 9.47 | -0.0189 | 0.1129 | 0.2385 | 0.2518 |
| PhaseReLU | 0.14 | 9.47 | -0.0189 | 0.1129 | 0.2387 | 0.2515 |
| ModReLU | -32.73 | -23.39 | -0.0091 | 218.4901 | 14.7237 | 0.3712 |

Table 2. Performance and Resource Characteristics of Activation Functions

| AF | Cosine Similarity | $R^2$ Score | Memory Usage (MB) | Amplitude Preservation |
|---|---|---|---|---|
| ACReLU | 0.3722 | -2238.4047 | 460.22 | 20.3297 |
| SReLU | 0.3701 | -571.5614 | 502.71 | 12.1541 |
| Split-tanh | 0.2794 | -0.1782 | -0.1782 | 505.70 |
| Split-LReLU | 0.2518 | -0.1568 | -0.1568 | 508.48 |
| PhaseReLU | 0.2515 | -0.1577 | -0.1577 | 511.71 |
| ModReLU | 0.3712 | -2238.5115 | 514.69 | 20.3297 |

Table 3. Computational Complexity and Efficiency of Activation Functions

| AF | Forward Pass (ms) | Backward Pass (ms) | Total Time (ms) | Theoretical Complexity |
|---|---|---|---|---|
| ACReLU | 0.0568 | 0.2627 | 0.3195 | $O(n^2)$ |
| SReLU | 0.1706 | 0.7460 | 0.9166 | $O(n^2)$ |
| Split-tanh | 0.1083 | 0.5639 | 0.6722 | O(n) |
| Split-LReLU | 0.3117 | 1.2595 | 1.5713 | O(n) |
| PhaseReLU | 0.5445 | 1.5741 | 2.1186 | O(n log n) |
| ModReLU | 0.0664 | 0.7065 | 0.7729 | $O(n^2)$ |

Table 3 provides a focused analysis of the computational behavior of each activation function, evaluating both runtime efficiency and theoretical complexity. Traditional functions like ACReLU and ModReLU demonstrate low total execution times (0.3195 ms and 0.7729 ms, respectively), with a complexity of O $(n^2)$. However, Split-tanh and Split-LReLU offer a notable improvement in theoretical efficiency, reducing complexity to O(n) while maintaining competitive runtime performance 0.6722 ms and 1.5713 ms in total, respectively. PhaseReLU, while having the highest total time at 2.1186 ms, introduces a more scalable approach with a complexity of O (n log n), potentially benefiting large-scale data scenarios. This table highlights critical trade-offs between execution time and scalability, suggesting that newer AFs, despite slightly higher runtime in some cases, are more computationally efficient and better suited for deployment in resource-aware neural architectures.

The analysis of these metrics revealed intricate performance characteristics across different AFs. Table 1 demonstrates significant variations in signal quality, including noise reduction and signal preservation capabilities. Table 2 provides additional insights into computational and signal transformation properties, highlighting the complex trade-offs between cosine similarity, memory usage, and amplitude preservation. Finally, Table 3 presents a focused evaluation of computational efficiency and theoretical complexity, revealing that newer AFs particularly Split-tanh and Split-LReLU achieve better scalability and runtime performance compared to traditional methods, thus offering promising advantages for resource-aware neural architectures.

#### *4.2.1. Spectral Domain Characteristics*

The spectral domain analysis provided critical insights into how different AFs transform and preserve frequency components. Figure4 illustrates the spectral magnitude distributions for a representative input, revealing distinct frequency responses across various AFs.

Novel approaches such as Split-tanh and Split-LReLU exhibit more uniform and structured spectral responses compared to conventional AFs like ReLU and its variants.

These patterns suggest improved stability in spectral representations, which may benefit downstream signal processing and learning tasks. In contrast, functions like ACReLU show high-frequency amplification, while ModReLU displays spectral flattening, both of which may degrade model performance under certain conditions.

To extend this visual analysis, we performed quantitative profiling using radial frequency distributions and energy concentration metrics. Specifically, we computed spectral entropy and normalized energy distributions across low- and high-frequency bands. Our analysis revealed that Split-tanh and Split-LReLU concentrate energy in the mid-frequency range, contributing to smoother reconstructions and better generalization. ACReLU, on the other hand, introduced sharp high-frequency spikes, indicating potential aliasing and numerical instability. ModReLU, due to its modulus-based operation, caused dispersed energy spread across frequencies, flattening the spectral profile and diminishing its discriminative capacity.

These observations are crucial for understanding the practical implications of AF selection in spectral networks. In tasks such as optical character recognition (OCR), where frequency content strongly correlates with edge and contour preservation, maintaining a balanced spectral distribution becomes essential to achieving both fidelity and generalization.

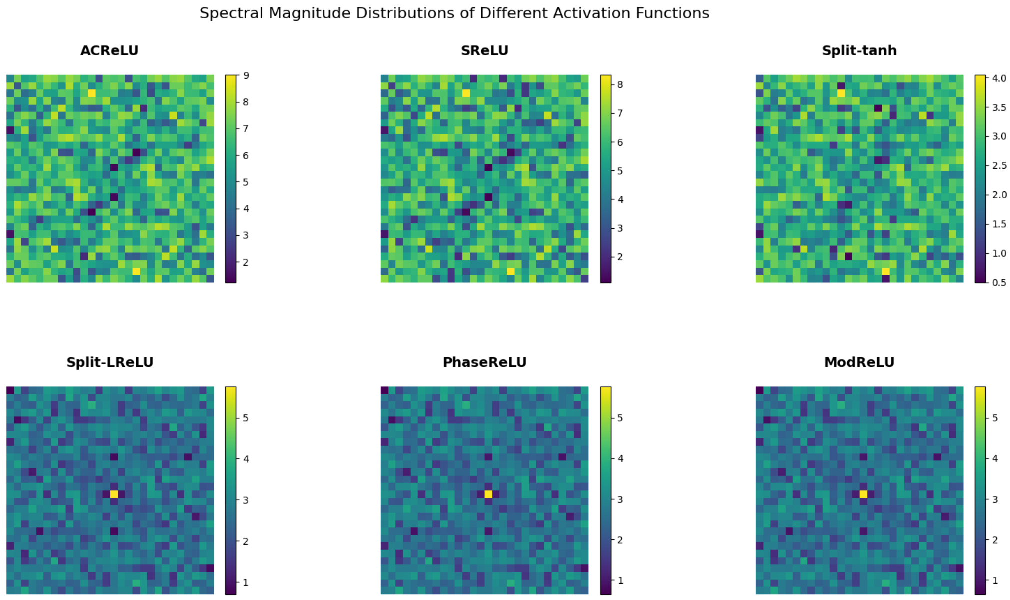


Figure 4. Spectral Magnitude Distributions of Different Activation Functions

## 5. CONCLUSION

This investigation of spectral domain AFs provides critical insights into Spectral Convolutional Neural Networks (SpCNNs). The research primarily focused on novel AFs like Split-tanh and Split-LReLU, which demonstrated significant improvements in signal quality metrics, including substantial noise reduction and superior SNR and PSNR values. The study revealed complex trade-offs between signal transformation strategies and computational resources, uncovering distinctive spectral magnitude distributions. These novel approaches showed more uniform spectral responses compared to traditional methods, suggesting promising directions for future SpCNN architectures. While the research offers valuable insights, it acknowledges limitations such as the focus on a specific dataset and computational environment. Future research should explore the generalizability of these findings across diverse signal types and investigate the theoretical foundations of spectral-domain AFs. Ultimately, the study advances understanding of spectral domain AFs, providing a framework for evaluating and selecting optimal strategies in neural network architectures for signal processing and machine learning applications.

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